

Lagrange - Számítási feladat

i	0	1	2	3
$\vec{p}_i$	$\begin{bmatrix} 2 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\begin{bmatrix} -5 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 6 \\ 4 \end{bmatrix}$
u_i	1	3	5	6

Lagrange interpolációs polinomok

$$L_0^3(u) = \frac{(u-u_1)(u-u_2)(u-u_3)}{(u_0-u_1)(u_0-u_2)(u_0-u_3)} = \frac{(u-3)(u-5)(u-6)}{(1-3)(1-5)(1-6)} = \frac{u^3 - 14u^2 + 63u - 90}{-40}$$

$$(u^3 - 8u + 15)(u-6) = u^3 - 8u^2 + 15u - 6u^2 + 48u - 90 = u^3 - 14u^2 + 63u - 90$$

$$L_1^3(u) = \frac{(u-u_0)(u-u_2)(u-u_3)}{(u_1-u_0)(u_1-u_2)(u_1-u_3)} = \frac{(u-1)(u-5)(u-6)}{(3-1)(3-5)(3-6)} = \frac{u^3 - 12u^2 + 41u - 30}{12}$$

$$(u^2 - 6u + 5)(u-6) = u^3 - 6u^2 + 5u - 6u^2 + 36u - 30 = u^3 - 12u^2 + 41u - 30$$

$$L_2^3(u) = \frac{(u-u_0)(u-u_1)(u-u_3)}{(u_2-u_0)(u_2-u_1)(u_2-u_3)} = \frac{(u-1)(u-3)(u-6)}{(5-1)(5-3)(5-6)} = \frac{u^3 - 10u^2 + 27u - 18}{-8}$$

$$(u^2 - 4u + 3)(u-6) = u^3 - 4u^2 + 3u - 6u^2 + 24u - 18 = u^3 - 10u^2 + 27u - 18$$

$$L_3^3(u) = \frac{(u-u_0)(u-u_1)(u-u_2)}{(u_3-u_0)(u_3-u_1)(u_3-u_2)} = \frac{(u-1)(u-3)(u-5)}{(6-1)(6-3)(6-5)} = \frac{u^3 - 9u^2 + 23u - 15}{15}$$

$$(u^2 - 4u + 3)(u-5) = u^3 - 4u^2 + 3u - 5u^2 + 20u - 15 = u^3 - 9u^2 + 23u - 15$$

A Lagrange interpolációs görbe

$$\vec{p}(u) = \sum_{i=0}^3 \vec{p}_i L_i^3(u) = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \frac{u^3 - 14u^2 + 63u - 90}{-40} + \begin{bmatrix} 3 \\ 3 \end{bmatrix} \frac{u^3 - 12u^2 + 41u - 30}{12} + \begin{bmatrix} -5 \\ 2 \end{bmatrix} \frac{u^3 - 10u^2 + 27u - 18}{-8} + \begin{bmatrix} 6 \\ 4 \end{bmatrix} \frac{u^3 - 9u^2 + 23u - 15}{15}$$

$$\vec{p}(u) = \begin{bmatrix} \frac{49}{40}u^3 - \frac{243}{20}u^2 + \frac{1327}{40}u - \frac{81}{4} \\ \frac{7}{24}u^3 - \frac{13}{4}u^2 + \frac{269}{24}u - \frac{37}{4} \end{bmatrix}; \quad \dot{\vec{p}}(u) = \begin{bmatrix} \frac{147}{40}u^2 - \frac{243}{10}u + \frac{1327}{40} \\ \frac{7}{8}u^2 - \frac{13}{2}u + \frac{269}{24} \end{bmatrix}$$

A pontbeli érintővektorok

$$\dot{\vec{p}}(1) = \begin{bmatrix} \frac{251}{20} \\ \frac{67}{12} \end{bmatrix}; \quad \dot{\vec{p}}(3) = \begin{bmatrix} -\frac{133}{20} \\ -\frac{5}{12} \end{bmatrix}; \quad \dot{\vec{p}}(5) = \begin{bmatrix} \frac{71}{20} \\ \frac{7}{12} \end{bmatrix}; \quad \dot{\vec{p}}(6) = \begin{bmatrix} \frac{787}{40} \\ \frac{89}{24} \end{bmatrix}$$

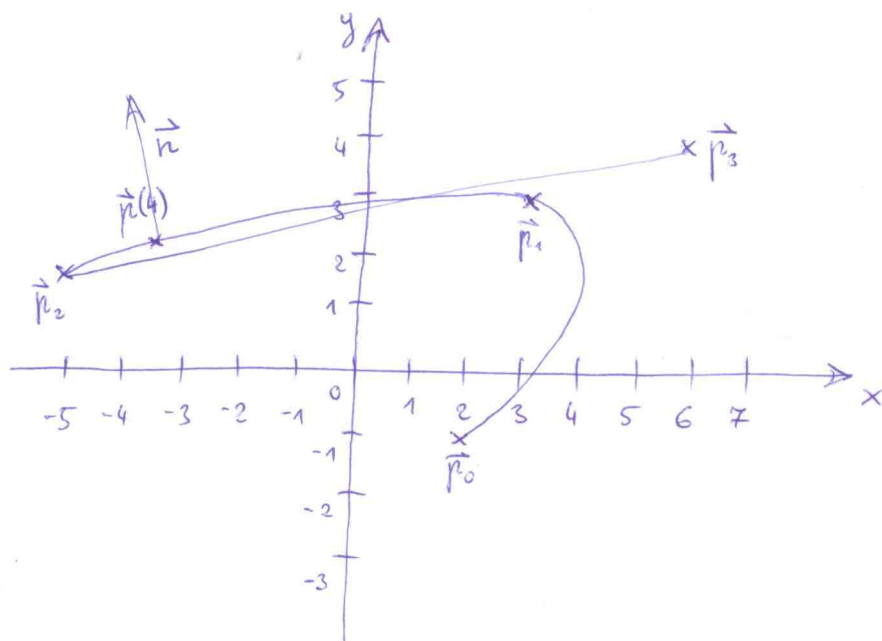
Az $u=4$ paraméterhez tartozó pont

$$\vec{p}(4) = \begin{bmatrix} -\frac{71}{20} \\ \frac{9}{4} \end{bmatrix} = \begin{bmatrix} -3.55 \\ 2.25 \end{bmatrix}; \quad \dot{\vec{p}}(4) = \begin{bmatrix} -\frac{209}{40} \\ -\frac{19}{24} \end{bmatrix} = \begin{bmatrix} -5.225 \\ -0.791\bar{6} \end{bmatrix}$$

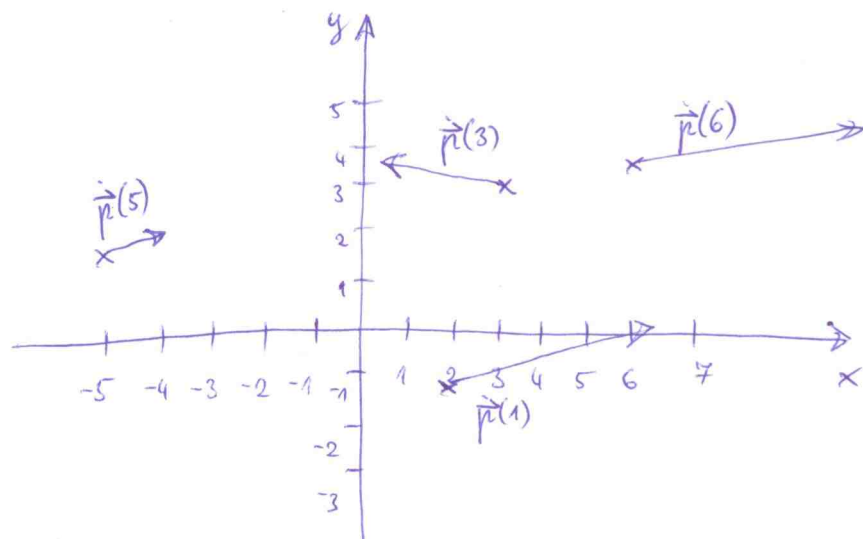
A ponthoz tartozó normálvektor

$$\vec{n} = \begin{bmatrix} -0.791\bar{6} \\ +5.225 \end{bmatrix}$$

Az interpolációs görbe ábrázolása



A pontokhoz tartozó érintővektorok



Catmull-Rom - Számítási feladat

i	0	1	2	3	4
$\vec{p}_i$	$\begin{bmatrix} 2 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\begin{bmatrix} -5 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 6 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 8 \\ 4 \end{bmatrix}$

Érintők számítása

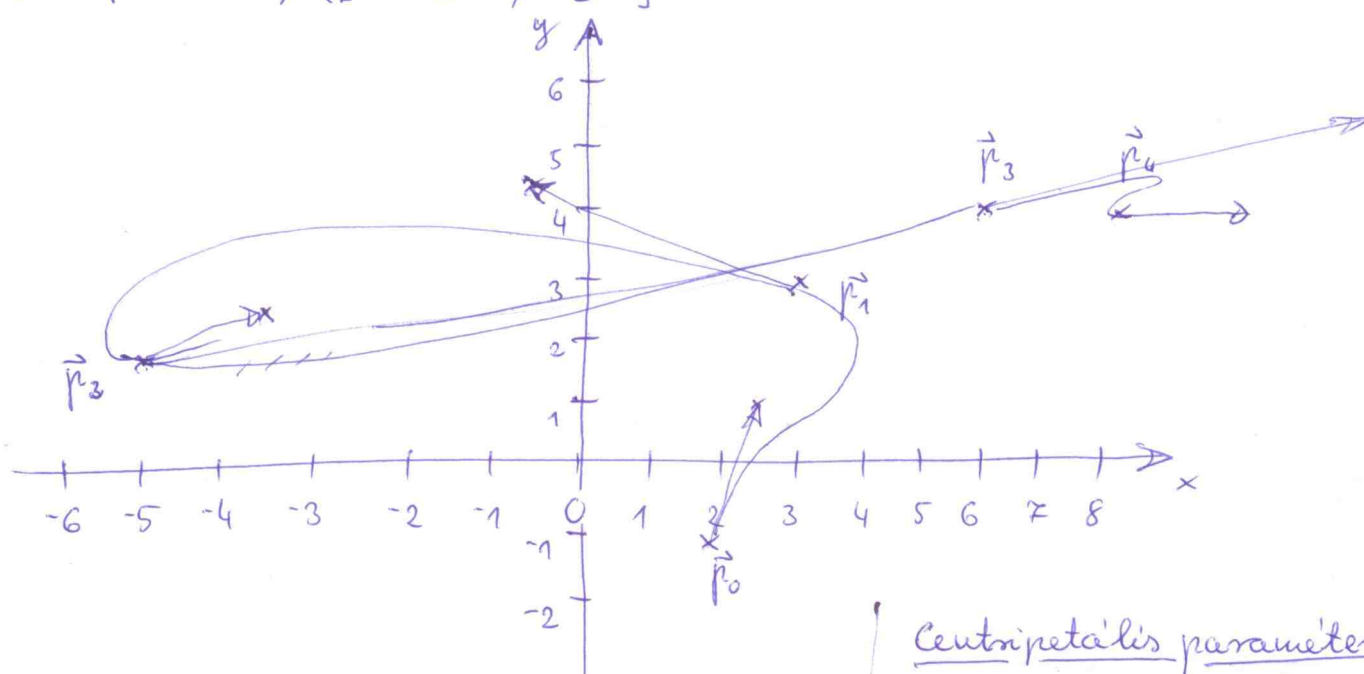
$$\vec{t}_0 = \tau(\vec{p}_1 - \vec{p}_0) = \frac{1}{2} \cdot \left(\begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}$$

$$\vec{t}_1 = \tau(\vec{p}_2 - \vec{p}_0) = \frac{1}{2} \cdot \left(\begin{bmatrix} -5 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} -3.5 \\ 1.5 \end{bmatrix}$$

$$\vec{t}_2 = \tau(\vec{p}_3 - \vec{p}_1) = \frac{1}{2} \cdot \left(\begin{bmatrix} 6 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}$$

$$\vec{t}_3 = \tau(\vec{p}_4 - \vec{p}_2) = \frac{1}{2} \cdot \left(\begin{bmatrix} 8 \\ 4 \end{bmatrix} - \begin{bmatrix} -5 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 6.5 \\ 1 \end{bmatrix}$$

$$\vec{t}_4 = \tau(\vec{p}_4 - \vec{p}_3) = \frac{1}{2} \cdot \left(\begin{bmatrix} 8 \\ 4 \end{bmatrix} - \begin{bmatrix} 6 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



Egyenlőtű paraméterezés

$$u_0=0, u_1=1, u_2=2, u_3=3, u_4=4$$

vagy

$$u_0=0, u_1=\frac{1}{4}, u_2=\frac{1}{2}, u_3=\frac{3}{4}, u_4=1$$

Húrközzel arányos paraméterezés

i	$\ \vec{p}_i - \vec{p}_{i-1}\ $	u_i
1	$\sqrt{17} \approx 4.1231$	0.1625
2	$\sqrt{65} \approx 8.0623$	0.4804
3	$\sqrt{125} \approx 11.18$	0.9212
4	2	1
$\Sigma \approx 25.366$		

paraméterez:

$$u_0=0$$

$$u_1=0.1635$$

$$u_2=0.4804$$

$$u_3=0.9212$$

$$u_4=1$$

Centripetalis paraméterezés

i	$\sqrt{\ \vec{p}_i - \vec{p}_{i-1}\ }$	u_i
1	$4\sqrt{17} \approx 2.0305$	0.2109
2	$4\sqrt{65} \approx 2.8394$	0.5058
3	$4\sqrt{125} \approx 3.3437$	0.8531
4	$4\sqrt{4} \approx 1.4142$	1
$\Sigma \approx 9.6279$		

paraméterez:

$$u_0=0$$

$$u_1=0.2109$$

$$u_2=0.5058$$

$$u_3=0.8531$$

$$u_4=1$$

Overhauser - Számítási feladat

i	0	1	2	3
$\vec{p}_i$	$\begin{bmatrix} 2 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 5 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 6 \\ -4 \end{bmatrix}$
u_i	1	3	5	6

A $\vec{c}_1(u)$ Bessel parabola számítása

$$L_0^2(u) = \frac{(u-u_1)(u-u_2)}{(u_0-u_1)(u_0-u_2)} = \frac{(u-3)(u-5)}{(1-3)(1-5)}$$

$$L_1^2(u) = \frac{(u-u_0)(u-u_2)}{(u_1-u_0)(u_1-u_2)} = \frac{(u-1)(u-5)}{(3-1)(3-5)}$$

$$L_2^2(u) = \frac{(u-u_0)(u-u_1)}{(u_2-u_0)(u_2-u_1)} = \frac{(u-1)(u-3)}{(5-1)(5-3)}$$

$$\vec{c}_1(u) = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \frac{u^2-8u+15}{8} + \begin{bmatrix} 3 \\ 3 \end{bmatrix} \frac{u^2-6u+5}{-4} + \begin{bmatrix} 5 \\ 2 \end{bmatrix} \frac{u^2-4u+3}{8}$$

$$\vec{c}_1(u) = \begin{bmatrix} \left(\frac{2}{8} - \frac{3}{4} + \frac{5}{8}\right)u^2 + \left(-\frac{16}{8} + \frac{18}{4} - \frac{20}{8}\right)u + \left(\frac{30}{8} - \frac{15}{4} + \frac{15}{8}\right) \\ \left(-\frac{1}{8} - \frac{3}{4} + \frac{2}{8}\right)u^2 + \left(1 + \frac{18}{4} - 1\right)u + \left(-\frac{15}{8} - \frac{15}{4} + \frac{6}{8}\right) \end{bmatrix} = \begin{bmatrix} 0.125u^2 + 1.875 \\ -0.625u^2 + 4.5u - 4.875 \end{bmatrix}$$

A $\vec{c}_2(u)$ Bessel parabola számítása

$$L_0^2(u) = \frac{(u-u_2)(u-u_3)}{(u_1-u_2)(u_1-u_3)} = \frac{(u-5)(u-6)}{(3-5)(3-6)}$$

$$L_1^2(u) = \frac{(u-u_1)(u-u_3)}{(u_2-u_1)(u_2-u_3)} = \frac{(u-3)(u-6)}{(5-3)(5-6)}$$

$$L_2^2(u) = \frac{(u-u_1)(u-u_2)}{(u_3-u_1)(u_3-u_2)} = \frac{(u-3)(u-5)}{(6-3)(6-5)}$$

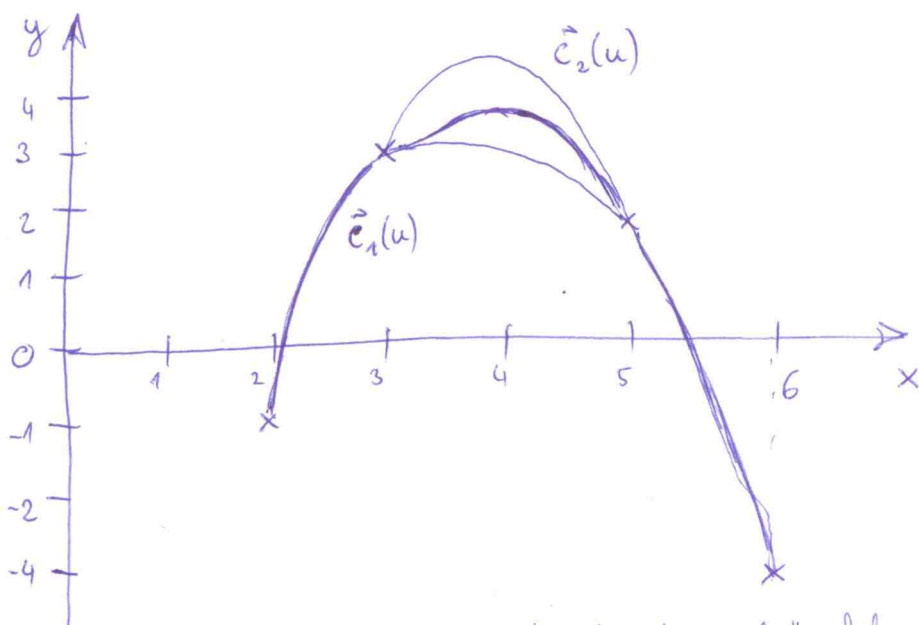
$$\vec{c}_2(u) = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \frac{u^2-11u+30}{6} + \begin{bmatrix} 5 \\ 2 \end{bmatrix} \frac{u^2-9u+18}{-2} + \begin{bmatrix} 6 \\ -4 \end{bmatrix} \frac{u^2-8u+15}{3}$$

$$\vec{c}_2(u) = \begin{bmatrix} \left(\frac{3}{6} - \frac{5}{2} + \frac{6}{3}\right)u^2 + \left(-\frac{33}{6} + \frac{45}{2} - \frac{48}{3}\right)u + \left(\frac{90}{6} - \frac{90}{2} + \frac{90}{3}\right) \\ \left(\frac{3}{6} - 1 - \frac{4}{3}\right)u^2 + \left(-\frac{33}{6} + 9 + \frac{32}{3}\right)u + \left(\frac{90}{6} - 18 - 20\right) \end{bmatrix} = \begin{bmatrix} u \\ -1.83u^2 + 14.16u - 23 \end{bmatrix}$$

A spline paraméteres alakja

$$\vec{c}(u) = \begin{cases} \vec{c}_1(u), & \text{ha } 1 \leq u < 3, \\ \left(1 - \frac{5-u}{2}\right) \vec{c}_1(u) + \left(\frac{5-u}{2}\right) \vec{c}_2(u), & \text{ha } 3 \leq u < 5, \\ \vec{c}_2(u), & \text{ha } 5 \leq u \leq 6. \end{cases}$$

A görbe ábrázolása



A görbe paraboláinak deriváltjai

$$\vec{c}_1(u) = \begin{bmatrix} 0.25u \\ -1.25u + 4.5 \end{bmatrix}$$

$$\vec{c}_2(u) = \begin{bmatrix} 1 \\ -\frac{11}{3}u + \frac{85}{6} \end{bmatrix}$$

Az interpolációs pontokhoz tartozó érintővektorok

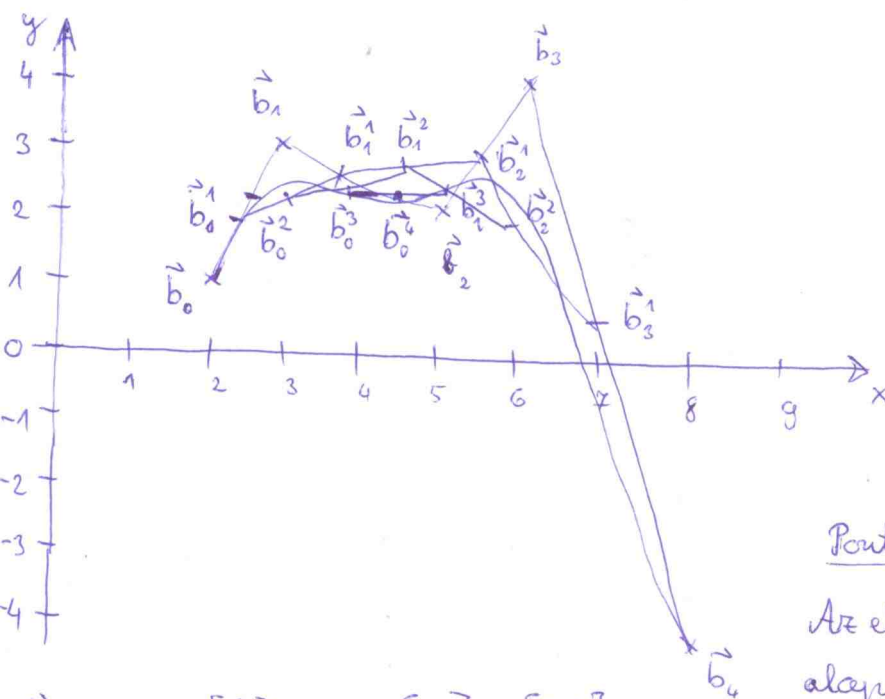
$$\vec{c}(1) = \vec{c}_1(1) = \begin{bmatrix} \frac{1}{4} \\ \frac{13}{4} \end{bmatrix}; \quad \vec{c}(3) = \vec{c}_1(3) = \begin{bmatrix} \frac{3}{4} \\ \frac{3}{4} \end{bmatrix}$$

$$\vec{c}(5) = \vec{c}_2(5) = \begin{bmatrix} 1 \\ -\frac{25}{6} \end{bmatrix}, \quad \vec{c}(6) = \vec{c}_2(6) = \begin{bmatrix} 1 \\ -\frac{47}{6} \end{bmatrix}$$

A görbe pontja az $u=4$ paraméternél

$$\vec{c}(4) = \frac{1}{2} \vec{c}_1(4) + \frac{1}{2} \vec{c}_2(4) = \frac{1}{2} \cdot \begin{bmatrix} 1 \\ -\frac{25}{6} \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} 4 \\ -\frac{47}{6} \end{bmatrix} = \begin{bmatrix} 2.5 \\ -\frac{36}{6} \end{bmatrix} = \begin{bmatrix} 2.5 \\ -6 \end{bmatrix}$$

de Casteljau - Számítási feladat



$$\vec{b}_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\vec{b}_1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\vec{b}_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\vec{b}_3 = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$\vec{b}_4 = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

Pontbeli érintő felírása

Az érintő irányát a $\vec{b}_0^3$ és $\vec{b}_1^3$ pontok alapján ismerjük.

$$\vec{t} = \vec{b}_1^3 - \vec{b}_0^3 = \begin{bmatrix} 1.496 \\ 0.064 \end{bmatrix}$$

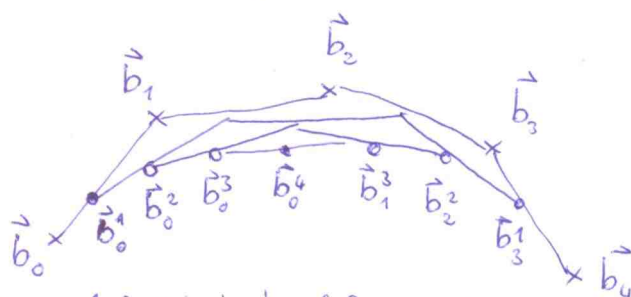
Az érintő egyenlete:

$$\vec{e}(\lambda) = \begin{bmatrix} 4.1504 \\ 2.3696 \end{bmatrix} + \lambda \cdot \begin{bmatrix} 1.496 \\ 0.064 \end{bmatrix}, \lambda \in \mathbb{R}$$

A pontbeli normálvektor

$$\vec{n} = \begin{bmatrix} 0.064 \\ -1.496 \end{bmatrix}$$

Görbe kettévágása



A kettévágással kapott kontrollpontok:

$$\vec{b}_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\vec{b}_4 = \begin{bmatrix} 4.1504 \\ 2.3696 \end{bmatrix}$$

$$\vec{b}_1 = \begin{bmatrix} 2.4 \\ 1.8 \end{bmatrix}$$

$$\vec{b}_1^3 = \begin{bmatrix} 5.048 \\ 2.408 \end{bmatrix}$$

$$\vec{b}_2 = \begin{bmatrix} 2.96 \\ 2.12 \end{bmatrix}$$

$$\vec{b}_2^2 = \begin{bmatrix} 5.96 \\ 2 \end{bmatrix}$$

$$\vec{b}_3 = \begin{bmatrix} 3.552 \\ 2.344 \end{bmatrix}$$

$$\vec{b}_3^1 = \begin{bmatrix} 6.8 \\ 0.8 \end{bmatrix}$$

$$\vec{b}_4 = \begin{bmatrix} 4.1504 \\ 2.3696 \end{bmatrix}$$

$$\vec{b}_4 = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

$$\vec{b}_0^1 = 0.6 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2.4 \\ 1.8 \end{bmatrix}$$

$$\vec{b}_1^1 = 0.6 \cdot \begin{bmatrix} 3 \\ 3 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3.8 \\ 2.6 \end{bmatrix}$$

$$\vec{b}_2^1 = 0.6 \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 5.4 \\ 2.8 \end{bmatrix}$$

$$\vec{b}_3^1 = 0.6 \cdot \begin{bmatrix} 6 \\ 4 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 8 \\ -4 \end{bmatrix} = \begin{bmatrix} 6.8 \\ 0.8 \end{bmatrix}$$

$$\vec{b}_0^2 = 0.6 \cdot \begin{bmatrix} 2.4 \\ 1.8 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 3.8 \\ 2.6 \end{bmatrix} = \begin{bmatrix} 2.96 \\ 2.12 \end{bmatrix}$$

$$\vec{b}_1^2 = 0.6 \cdot \begin{bmatrix} 3.8 \\ 2.6 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 5.4 \\ 2.8 \end{bmatrix} = \begin{bmatrix} 4.44 \\ 2.68 \end{bmatrix}$$

$$\vec{b}_2^2 = 0.6 \cdot \begin{bmatrix} 5.4 \\ 2.8 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 6.8 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 5.96 \\ 2 \end{bmatrix}$$

$$\vec{b}_0^3 = 0.6 \cdot \begin{bmatrix} 2.96 \\ 2.12 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 4.44 \\ 2.68 \end{bmatrix} = \begin{bmatrix} 3.552 \\ 2.344 \end{bmatrix}$$

$$\vec{b}_1^3 = 0.6 \cdot \begin{bmatrix} 4.44 \\ 2.68 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 5.96 \\ 2 \end{bmatrix} = \begin{bmatrix} 5.048 \\ 2.408 \end{bmatrix}$$

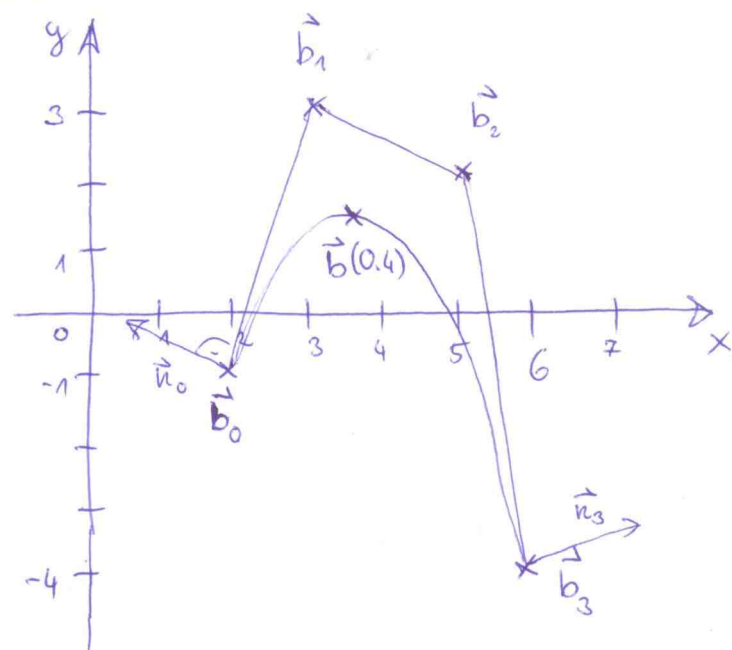
$$\vec{b}_0^4 = 0.6 \cdot \begin{bmatrix} 3.552 \\ 2.344 \end{bmatrix} + 0.4 \cdot \begin{bmatrix} 5.048 \\ 2.408 \end{bmatrix} = \begin{bmatrix} 4.1504 \\ 2.3696 \end{bmatrix}$$

$$\vec{b}(0.4) = \begin{bmatrix} 4.1504 \\ 2.3696 \end{bmatrix}$$

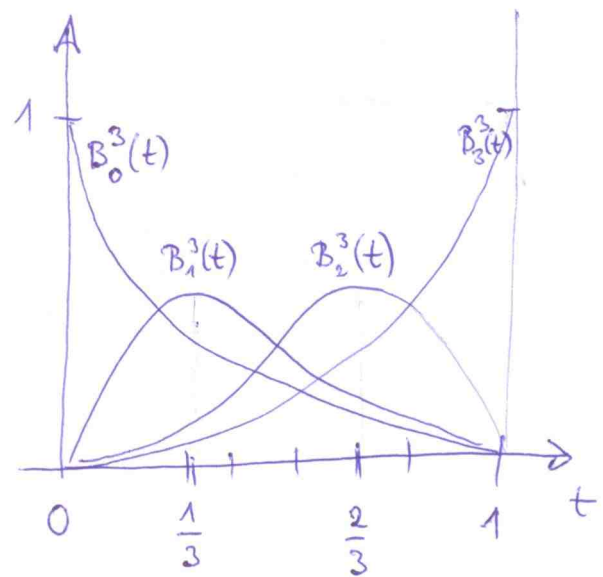
A Bézier görbe $t=0.4$ paraméterhez tartozó

pontja a $\begin{bmatrix} 4.1504 \\ 2.3696 \end{bmatrix}$.

Bézier - Számítási feladat



Bernstein polinomok ábrázolása



Bernstein polinomok

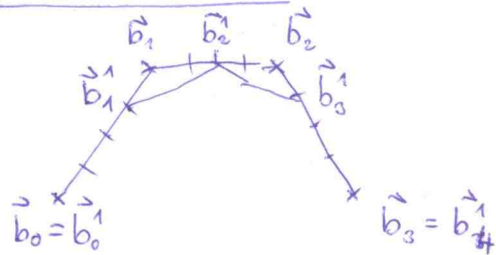
$$B_0^3(0.4) = \binom{3}{0} 0.4^0 \cdot 0.6^3 = 0.216$$

$$B_1^3(0.4) = \binom{3}{1} 0.4^1 \cdot 0.6^2 = 0.432$$

$$B_2^3(0.4) = \binom{3}{2} 0.4^2 \cdot 0.6^1 = 0.288$$

$$B_3^3(0.4) = \binom{3}{3} 0.4^3 \cdot 0.6^0 = 0.064$$

Fokszámnövelés



$$\vec{b}_0^1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\vec{b}_1^1 = \frac{1}{4} \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} + \frac{3}{4} \cdot \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2.75 \\ 2 \end{bmatrix}$$

$$\vec{b}_2^1 = \frac{1}{2} \cdot \begin{bmatrix} 3 \\ 3 \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2.5 \end{bmatrix}$$

$$\vec{b}_3^1 = \frac{3}{4} \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} + \frac{1}{4} \cdot \begin{bmatrix} 6 \\ -4 \end{bmatrix} = \begin{bmatrix} 5.25 \\ 0.5 \end{bmatrix}$$

$$\vec{b}_4^1 = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

$$\vec{b}(0.4) = 0.216 \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 0.432 \cdot \begin{bmatrix} 3 \\ 3 \end{bmatrix} + 0.288 \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} + 0.064 \cdot \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

$$\vec{b}(0.4) = \begin{bmatrix} 3.552 \\ 1.4 \end{bmatrix}$$

Normálvektorok a végpontokban

$$\vec{t}_0 = \vec{b}_1 - \vec{b}_0 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\vec{n}_0 = \frac{1}{\sqrt{17}} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} \approx \begin{bmatrix} -0.9701 \\ 0.2425 \end{bmatrix}$$

$$\vec{t}_3 = \vec{b}_3 - \vec{b}_2 = \begin{bmatrix} 6 \\ -4 \end{bmatrix} - \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \end{bmatrix}$$

$$\vec{n}_3 = \frac{1}{\sqrt{37}} \cdot \begin{bmatrix} 6 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 0.9864 \\ 0.1644 \end{bmatrix}$$

A görbe kezdőpontjához az $\vec{n}_0$, végpontjához pedig az $\vec{n}_3$ normálvektorokat adhatjuk meg.

A fokszámnövelést követően tehát a

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2.75 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2.5 \end{bmatrix}, \begin{bmatrix} 5.25 \\ 0.5 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

kontrollpontokat kapjuk.