DETERMINATION OF JOINT PARAMETERS FOR A GIVEN TOOLPATH USING INVERSE KINEMATICS

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ABSTRACT

The aim of this study is to describe the inverse kinematics of the examined structure in the case of a specified toolpath. The specified toolpath is the Viviani curve. The Viviani curve was chosen for the path of the tool because, due to the construction of the analysed structure, the end point of the tool can traverse the points of a spherical surface. The Viviani curve is created by the intersection of the sphere and the cylinder. Thus, the end point of the tool can follow the path defined by the given curve. The joint parameters required to travel on the specified track will be described.

1. INTRODUCTION

The second chapter presents the construction of the examined structure and mechanism. Then the toolpath will be described. The fourth chapter examines the equation of the toolpath. After that, the fifth chapter explains the inverse kinematics of the central universal joint [1]. The study continues by checking the inverse kinematics of the central universal joint, where the results obtained by using two software are compared. The seventh chapter presents the inverse kinematics of the actuator [2]. Finally, the eighth chapter summarises the tasks performed in the study.

2. TILTING TABLE STRUCTURE

The mechanism includes seven universal joints and three prismatic joints that connect the tilting table to the base. It can be seen in Fig. 1 that three actuators have the same structure. Universal joint, prismatic joint, and universal joint. The central connection consists of a universal joint.

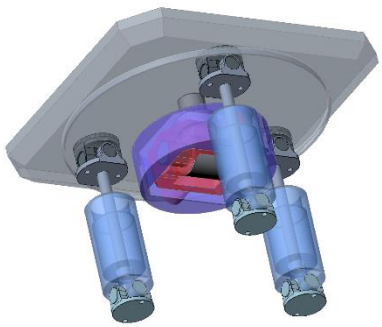


Fig. 1.
Tilting table

3. TOOLPATH

Toolpath is introduced in this chapter. Toolpath in Fig. 2 is given by the intersection of a sphere and cylinder, what is a Viviani curve [3]. The sphere is marked with a grid surface. The cylinder is green. The Viviani curve is blue.

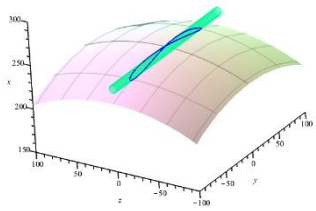


Fig. 2.
Toolpath calculated with Maple

The Viviani curve is red in the Fig. 3. The tool point is moving along the red curve. The Viviani curve is enlarged in Fig. 4.

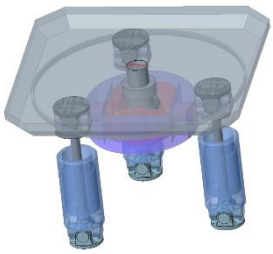


Fig. 3.
Toolpath created by Creo

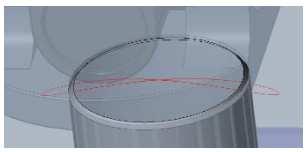


Fig. 4.
Toolpath

4. EQUATION OF TOOLPATH

The Viviani curve can be seen in Fig. 5. The equation of sphere is in equation (1). The equation of a cylinder is in equation (2). The (3)–(5) equations show the parametric equation of the coordinates of the Viviani curve [6] takes values in the range of 0 to 4 pi.

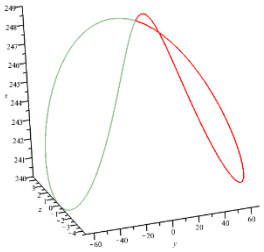


Fig. 5.
Viviani curve

$$x^2 + y^2 + z^2 = a^2 \tag{1}$$

$$x^2 + (z - c)^2 = b^2 \tag{2}$$

$$x = b \cdot \cos(t) \tag{3}$$

$$y = \sqrt{a^2 - b^2 - c^2 - 2bc \cdot \sin(t)} \tag{4}$$

$$z = c + b \cdot \sin(t) \tag{5}$$

5. INVERSE KINEMATICS OF CENTRAL UNIVERSAL JOINT

A direct kinematic equation can be seen in equation (6) [7]. The direct kinematic equation is given by the product of the direct kinematic function, equation (7), and the vector p, in equation (8) [8]. Vector p, equation (8) is the position vector of the starting point of the path. The first row of equation (6) is the x coordinate. The second row of equation (6) is the y coordinate. The third row of equation (6) is the z coordinate. Equation (6) yields θ_1, equation (10) and θ_2, equation (9).

$$DKF \cdot p = \begin{bmatrix} 249cos(\theta_1)cos(\theta_2) \\ 249sin(\theta_1)cos(\theta_2) \\ 249sin(\theta_2) \end{bmatrix} \tag{6}$$

$$DKF = \begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & \sin(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & -\cos(\theta_1) \\ sin\theta_2 & cos\theta_2 & 0 \end{bmatrix} \tag{7}$$

$$p = \begin{bmatrix} 249 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

$$\theta_2 = arcsin\left(\frac{z}{249}\right) \tag{9}$$

$$\theta_1 = arccos\left(\frac{x}{\sqrt{-z^2+62001}}\right) \tag{10}$$

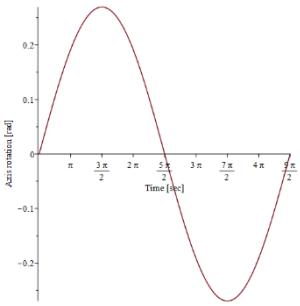


Fig. 6.
Function of θ_1

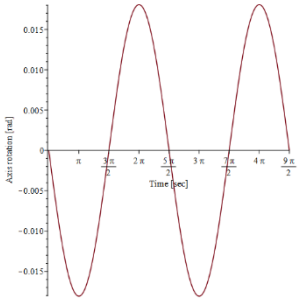


Fig. 7.
Function of θ_2

The θ_1 function in the range 0–4 pi is shown in Fig. 6. The θ_2 function in the range 0–4 pi is shown in Fig. 7.

6. CROSS – CHECKING INVERSE KINEMATICS OF CENTRAL UNIVERSAL JOINT

The model has been investigated with Creo, and the math model was calculated with Maple [9], [10]. Creo and Maple results are the same as the Fig. 8–10 shows.

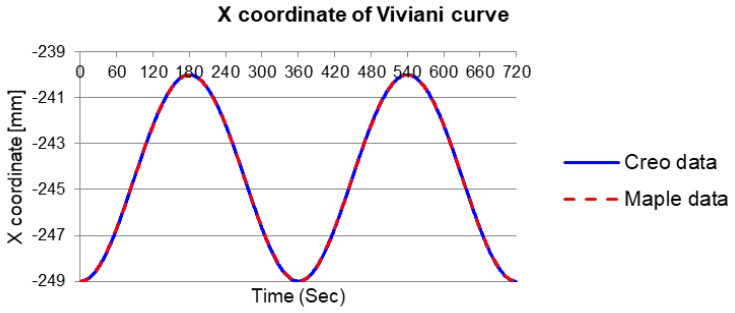


Fig. 8.
X coordinate of the Viviani curve

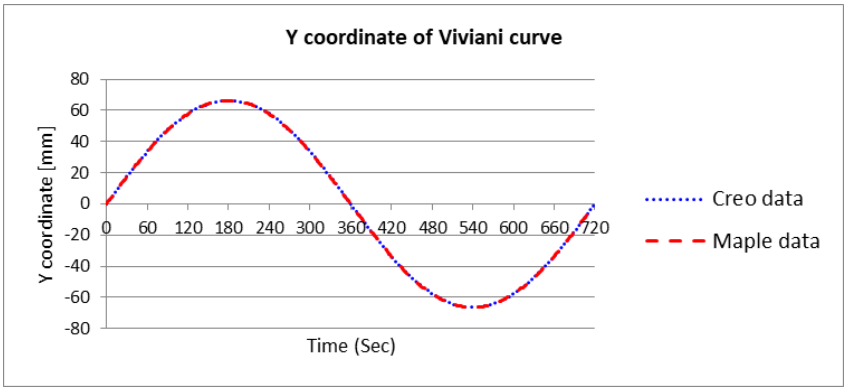


Fig. 9.
Y coordinate of the Viviani curve

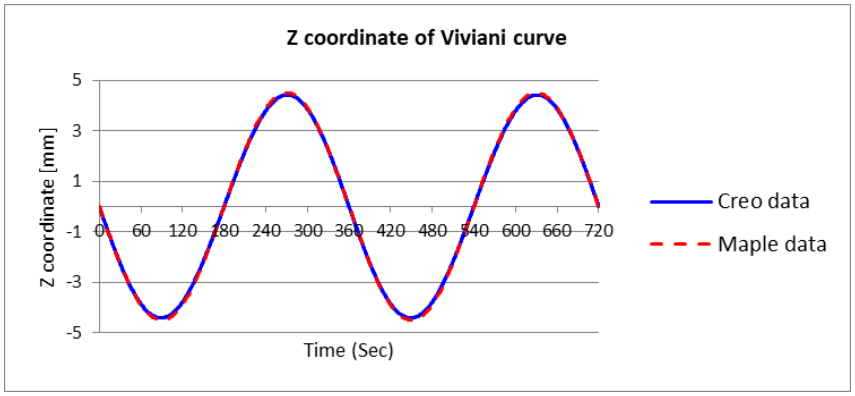


Fig. 10.
Z coordinate of the Viviani curve

7. INVERSE KINEMATICS OF ACTUATOR

The path of the tool and the path of the actuators can be seen in Fig. 11.

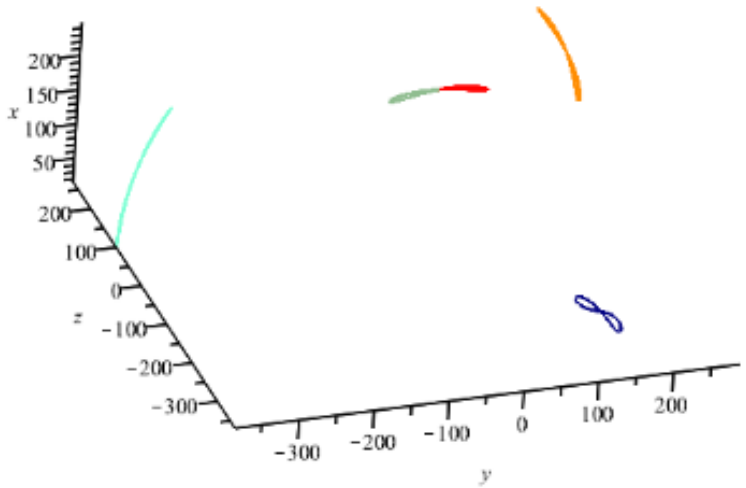


Fig. 11.
Path of tool and actuators

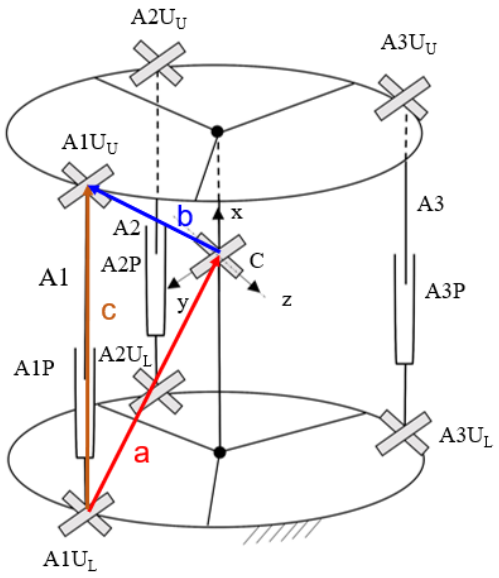


Fig. 12.
Vector representation of an inverse kinematics solution for an actuator

$$\vec{a} + \vec{b} = \vec{c} \tag{11}$$

Equation (11) can be read from Fig. 12, which gives the inverse kinematics of the actuator. $\vec{a}$ is constant. $\vec{b}$ is changing, but it can be calculated with equation (6).

The length of the actuator can be seen in Fig. 13. This is the solution to the inverse kinematics of actuators [2].

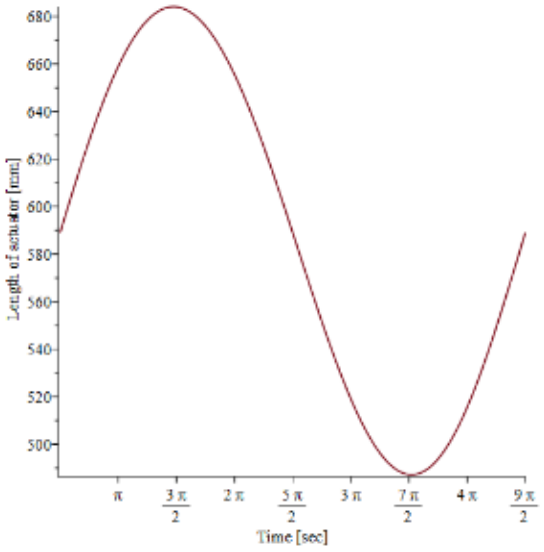


Fig. 13.
Inverse kinematics solution for actuator 1

8. SUMMARY

The purpose of this study is to present an investigation of the inverse kinematics of the structure. In the first step, it includes the structural design of the mechanism. Then the derivation of the toolpath was explained. Then, the parametric equations of the toolpath were derived. In the fifth chapter, starting from the direct kinematic equation, the solution of the inverse kinematics of the structure can be seen for the central universal joint. The sixth chapter contains a cross–check on the inverse kinematics of the central universal joint. Finally, the inverse kinematics of the actuator was illustrated.

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