$$\lim_{n\to\infty} \frac{3+n^2}{n^2} + \left(\frac{1}{n}\right)^2 = \lim_{n\to\infty} \overbrace{\frac{n^2}{n^2}}^{1} + \left(\frac{1}{n}\right)^2 = 1 + \left(\frac{1}{\infty}\right)^2 = 1 + 0^2 = \underline{\underline{1}}$$

$$\lim_{x\to\infty} \frac{2x^2+3x-7}{-x^2+5x+10} = \lim_{x\to\infty} \frac{2\cancel{x^2}}{-\cancel{x^2}} = \frac{2}{-1} = \underline{\underline{-2}}$$

$$\lim_{x\to-\infty} \frac{5x+10x^3+5}{4x^2+3x+2} = \lim_{x\to-\infty} \frac{10x^{\cancel{3}}}{4\cancel{x^2}} = \frac{10\cdot(-\infty)}{4} = \underline{\underline{-\infty}}$$

$$\lim_{n\to\infty} \left(4+\frac{1}{n}\right)^n = \lim_{n\to\infty} \left(4\left(1+\frac{\frac{1}{4}}{n}\right)\right)^n = \lim_{n\to\infty} \left(\underbrace{4^n}_{\infty} \cdot \underbrace{\left(1+\frac{\frac{1}{4}}{n}\right)^n}_{e^{\frac{1}{4}}}\right) =$$

$$= \infty \cdot e^{\frac{1}{4}} = \underline{\underline{\infty}}$$

$$\lim_{n\to\infty} \left(\frac{1}{4}+\frac{2}{n}\right)^n = \lim_{n\to\infty} \left(\frac{1}{4}\left(1+\frac{8}{n}\right)\right)^n = \lim \underbrace{\left(\frac{1}{4}\right)^n}_{0} \cdot \underbrace{\left(1+\frac{8}{n}\right)^n}_{e^8} =$$

$$= 0 \cdot e^8 = \underline{\underline{0}}$$

$$\lim_{x\to 0} \frac{\cos x^2 - 1}{e^x - 1} = \frac{\cos 0 - 1}{e^0 - 1} = \frac{1-1}{1-1} = \left(\frac{0}{0}\right) \overset{L'H}{=} \lim_{x\to 0} \frac{-\sin(x^2)\cdot 2x}{e^x} =$$

$$= \frac{-\sin(0^2)\cdot 2\cdot 0}{e^0} = \frac{0}{1} = \underline{\underline{0}}$$

$$\lim_{x\to 0} \frac{1-e^x}{\sin(2x)} = \frac{1-e^0}{\sin(0)} = \frac{1-1}{0} = \left(\frac{0}{0}\right) \overset{L'H}{=}$$

$$= \lim_{x\to 0} \frac{-e^x}{\cos(2x)\cdot 2} = \frac{-e^0}{2\cdot 1} = \boxed{-1} = \frac{-1}{2} \qquad (\cos(0)=1)$$

$$\lim_{n\to\infty}\frac{6-n^2}{n^2}+\left(\frac{6}{n}\right)^2=\lim_{n\to\infty}\underbrace{\frac{-\cancel{n^2}}{\cancel{n^2}}}_{-1}+\left(\frac{6}{n}\right)^2=-1+\left(\frac{6}{\infty}\right)^2=-1+0^2=\underline{\underline{-1}}$$

$$\lim_{x\to-\infty}\left(\frac{6}{x}\right)^3-\frac{12x^2+4}{3x^3+2}=\lim_{x\to-\infty}\left(\frac{6}{x}\right)^3-\frac{12\cancel{x^2}}{3x^{\cancel{3}}}=$$

$$=\left(\frac{6}{-\infty}\right)^3-\frac{12}{3(-\infty)}=0^3-0=\underline{\underline{0}}$$

$$\lim_{n\to\infty}\left(4+\frac{6}{n}\right)^n=\lim_{n\to\infty}\left(4\left(1+\frac{\frac{6}{4}}{n}\right)\right)^n=\lim_{n\to\infty}4^n\left(1+\frac{\frac{6}{4}}{n}\right)^n=$$

$$=\infty\cdot e^{\frac{6}{4}}=\underline{\underline{\infty}}$$

$$\lim_{x\to\infty}\left(-3+\frac{5}{n}\right)^{2n}=\lim_{x\to\infty}\left(-3\left(1+\frac{-\frac{5}{3}}{n}\right)\right)^{2n}=\cancel{\lim_{x\to\infty}(-3)^{2n}\left(1+\frac{-5}{}\right.}$$

$$=\lim_{x\to\infty}(-3)^{2n}\cdot\left(1+\frac{-\frac{5}{3}}{n}\right)^{2n}=\text{nem létezik}$$

$$\lim_{x\to0}\frac{\cos x-1}{e^{2x}-1}=\frac{\cos 0-1}{e^0-1}=\frac{1-1}{1-1}=\left(\frac{0}{0}\right)\overset{L'H}{=}\lim_{x\to0}\frac{-\sin x}{e^{2x}\cdot2}=$$

$$=\frac{-\sin 0}{e^0\cdot2}=\frac{0}{1\cdot2}=\underline{\underline{0}}$$

$$\lim_{n\to\infty}\frac{7-n^2}{n^3}+\left(\frac{7}{n}\right)^2=\lim_{n\to\infty}\left(\frac{-\cancel{n^2}}{n^{\cancel{3}}}\right)+\left(\frac{7}{n}\right)^2=\lim_{n\to\infty}\frac{-1}{n}+\left(\frac{7}{n}\right)^2=$$

$$=\frac{-1}{\infty}+\left(\frac{7}{\infty}\right)^2=0+0^2=\underline{\underline{0}}$$

$$\lim_{n\to\infty}\left(7+\frac{1}{n}\right)^n=\lim_{n\to\infty}\left(7\left(1+\frac{\frac{1}{7}}{n}\right)\right)^n=\lim_{n\to\infty}7^n\cdot\left(1+\frac{\frac{1}{7}}{n}\right)^n=\infty\cdot e^{\frac{1}{7}}=\underline{\underline{\infty}}$$

$$\lim_{x\to\infty}\left(-\frac{1}{7}+\frac{2}{x}\right)^{3x}=\lim_{x\to\infty}\left(-\frac{1}{7}\left(1+\frac{-14}{x}\right)\right)^{3x}=$$

$$=\lim_{x\to\infty}\left(-\frac{1}{7}\right)^{3x}\left(\left(1+\frac{-14}{x}\right)^x\right)^3=0\cdot\left(e^{-14}\right)^3=\underline{\underline{0}}$$

$$\lim_{x\to0}\frac{\sin x^2}{e^x-1}=\frac{\sin 0}{e^0-1}=\left(\frac{0}{0}\right)\overset{L'H}{=}\lim_{x\to0}\frac{\cos(x^2)\cdot 2x}{e^x}=$$

$$=\frac{\cos(0)\cdot2\cdot0}{e^0}=\frac{1\cdot0}{1}=\underline{\underline{0}}$$

$$\lim_{n\to\infty} \frac{5+n^2}{n} + \left(\frac{3}{n}\right)^2 = \lim_{n\to\infty} \frac{n^{\cancel{2}}}{\cancel{n}} + \left(\frac{3}{n}\right)^2 = \infty + \left(\frac{3}{\infty}\right)^2 = \infty + 0 = \underline{\underline{\infty}}$$

$$\lim_{n\to\infty}\left(1+\frac{6}{n}\right)^n = \underline{\underline{e^6}} \; ; \; \lim_{x\to\infty}\left(1-\frac{6}{x}\right)^x = \lim_{x\to\infty}\left(1+\frac{-6}{x}\right)^x = \underline{\underline{e^{-6}}}$$

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{5n} = \lim_{n\to\infty}\left(\left(1+\frac{2}{n}\right)^n\right)^5 = \left(e^2\right)^5 = \underline{\underline{e^{10}}}$$

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{-7x} = \lim_{x\to\infty}\left(\left(1+\frac{3}{x}\right)^x\right)^{-7} = \left(e^3\right)^{-7} = \underline{\underline{e^{-21}}}$$

$$\lim_{x\to 0}\frac{\sin x}{e^{2x}-1} = \frac{\sin 0}{e^0-1} = \left(\frac{0}{0}\right) \stackrel{L'H}{=} \lim_{x\to 0}\frac{\cos x}{e^{2x}\cdot 2} = \frac{\cos 0}{e^0\cdot 2} =$$

$$= \frac{1}{1\cdot 2} = \underline{\underline{\frac{1}{2}}}$$

# Határérték

$$\lim_{n\to\infty} (-1)^n \frac{5}{4n+1} = \begin{cases} \text{ha } n = 2k \ \text{(páros)} \quad k\in\mathbb{Z} \ \to +1\cdot\frac{5}{4n+1} \to \lim\limits_{n\to\infty} a_n = 0 \\ \text{ha } n = 2k+1 \ \text{(páratlan)} \quad k\in\mathbb{Z} \ \to (-1)\cdot\frac{5}{4n+1} \to \lim\limits_{n\to\infty} a_n = 0 \end{cases}$$

páros h.ér = páratlan h.ér. $\Rightarrow \exists$ h.ér. $= 0$

$$\lim_{n\to\infty}\left(\frac{3n+2}{3n}\right)^n = \lim_{n\to\infty}\left(\frac{3n}{3n}+\frac{2}{3n}\right)^n = \lim_{n\to\infty}\left(1+\frac{\frac{2}{3}}{n}\right)^n = e^{\frac{2}{3}}$$

$$\lim_{x\to 0}(x\cdot \operatorname{ctg} 2x) = (0\cdot +\infty) = \left(\frac{0}{0}\right) = \left(\frac{\infty}{\infty}\right) = \lim_{x\to 0}\frac{x}{\operatorname{tg} 2x} = \frac{0}{0} \overset{L'H.}{=}$$

$$\lim_{x\to 0}\frac{1}{\frac{1}{\cos^2 2x}\cdot 2} = \lim_{x\to 0}\frac{\cos^2 2x}{2} = \frac{1}{2}$$

$$\lim_{x\to 0}\frac{\operatorname{ctg} 2x}{\frac{1}{x}} = \left(\frac{\infty}{\infty}\right) \overset{L'H.}{=} \lim_{x\to 0}\frac{\frac{-1}{\sin^2 2x}\cdot 2}{\frac{-1}{x^2}} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to 0}\frac{+2x^2}{+\sin^2 2x} =$$

$$\overset{L'H}{=} \lim_{x\to 0}\frac{\cancel{2}\cdot 2x}{\cancel{2}(\sin 2x)(\cos 2x)\cdot 2} = \lim_{x\to 0}\frac{x}{(\sin 2x)(\cos 2x)} = \left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{1}{\left((\cos 2x)^2-(\sin 2x)^2\right)2}$$

$$= \frac{1}{2}$$

$$\lim_{x\to 0}\frac{e^{2x}-1}{\underbrace{x^3+6x}_{0}} = \frac{0}{0} \overset{L'H.}{=} \lim_{x\to 0}\frac{e^{2x}\cdot 2 - 0}{3x^2+6} = \frac{2}{6} = \frac{1}{3}$$

$$\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{100+2n} = \lim_{n\to\infty} \underbrace{\left(1+\frac{4}{n}\right)^{100}}_{\to 1} \cdot \underbrace{\left(1+\frac{4}{n}\right)^{2n}}_{(e^4)^2} = 1\cdot e^8 = \underline{\underline{e^8}}$$

$$\lim_{x\to 0}\frac{e^{2x}-1}{5\sin 4x} = \left(\frac{0}{0}\right) \overset{L'H}{=} \lim_{x\to 0}\frac{2\underbrace{e^{2x}}_{\to 1}}{5\cdot(\cos 4x)\cdot 4} = \frac{2\cdot 1}{20\cdot 1} = \underline{\underline{\frac{1}{10}}}$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x^2+x} = \left(\frac{0}{0}\right) \overset{L'H}{=} \lim_{x\to 0}\frac{\overbrace{\frac{1}{\cos^2 x}}^{\to 1}}{\underbrace{2x}_{\to 0}+\underbrace{1}_{\to 1}} = \frac{1}{1} = \underline{\underline{1}}$$