Probability and Mathematical Statistics

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Seminar 1

Sets, Combinatorics, Classical probability space

1. Combinatorics

Problem 1

How many different orders are the letters A, A, B, B, B, C, C?

The solution of Problem 1

$$P_7^{2,3,2(r)} = \frac{7!}{2! \cdot 3! \cdot 2!} = 210.$$

Problem 2

Seven guests arrive at a camping. They can be placed in 3 cabin, where the first has 2 beds, the second has 3 beds, and the third has 2 beds. How many ways can the guests be placed in the cabins if we only care about which guests are placed to which cabin?

Solution to Problem 2

Let the guests be labeled $1, 2, 3, 4, 5, 6, 7$, and let the cabins be labeled A, A, B, B, B, C, C (according to the number of beds). The following table shows one example of the guests' accommodation in the cabins:

1	2	3	4	5	6	7
B	A	B	C	A	C	C

Thus, based on the table, the number of ways to accommodate the guests in the cabins is

$$P_7^{2,3,2(i)} = \frac{7!}{2! \cdot 3! \cdot 2!} = 210.$$

Problem 3

How many ways can we place 4 identical letters into 5 numbered boxes if

a. at most one letter can be placed in each box;

b. more than one letter can be placed in a box.

Solution to Problem 3

4 identical letters into 5 labeled boxes:

a. $n = 5,\ k = 4,$
$$C_n^k = \binom{n}{k} = \binom{5}{4} = 5.$$

b. $n = 5,\ k = 4,$
$$C_n^{k(i)} = \binom{n+k-1}{k} = \binom{8}{4} = 70.$$

Problem 4

How many ways can we place 4 labeled letters into 5 labeled boxes if

a. at most one letter can be placed in each box;

b. more than one letter cane be placed in each box.

Solution to Problem 4

4 labeled letters into 5 labeled boxes:

a. $n = 5,\ k = 4,$
$V_n^k = (n)_k = n(n-1)\dots(n-k+1) = 5 \cdot 4 \cdot 3 \cdot 2 = 120.$

b. $n = 5,\ k = 4,$
$V_n^{k(i)} = 5^4 = 625.$

Problem 5

In a candy packaging factory, a machine fills small bags with 5 types of candies. Each bag is filled randomly with 4 candies. How many different types of bags can be produced during this process?

Solution to Problem 5

$n = 5, \ k = 4,$
$C_n^{k(i)} = \binom{n+k-1}{k} = \binom{8}{4} = 70.$

2. Classical probability space

Problem 6

An urn contains 6 balls labeled from 0 to 5. Without replacement, we draw 3 balls. What is the probability that the drawn numbers, in the order of the drawn, are the digits of

a. a 3-digit number (A event);

b. a 3-digit even number (B event);

c. a 3-digit odd number (C event).

Solution to Problem 6

Digits: $0, 1, 2, 3, 4, 5$.
Without replacement, we draw 3 balls.
$\#\Omega = V_6^3 = 6 \cdot 5 \cdot 4 = 120$. $\#A := 5 \cdot 5 \cdot 4 = 100$.
To determine $\#B$, note that $B = B_1 \cup B_2$ is a disjoint union, where
$B_1 := \{$the 3-digit even number obtained has an even first digit$\}$,
$B_2 := \{$the 3-digit even number obtained has an odd first digit$\}$.
$\#B_1 = 2 \cdot 4 \cdot 2 = 16$, since the first place can be either digit 2 or 4, the last place can be one of the remaining 2 even digits, and the middle place can be any of the remaining 4 digits.
$\#B_2 = 3 \cdot 4 \cdot 3 = 36$, by a similar argument as for $\#B_1$. Thus
$\#B = \#B_1 + \#B_2 = 16 + 36 = 52$.
Since $\#C = \#A - \#B$, we have $\#C = 100 - 52 = 48$.

Solution to Problem 6 (continued)

The required probabilities are

$$\mathbb{P}(A) = \frac{\#A}{\#\Omega} = \frac{100}{120} = \frac{5}{6} = 0.8333;$$
$$\mathbb{P}(B) = \frac{\#B}{\#\Omega} = \frac{52}{120} = \frac{13}{30} = 0.4333;$$
$$\mathbb{P}(C) = \frac{\#C}{\#\Omega} = \frac{48}{120} = \frac{2}{5} = 0.4;$$

Problem 7

An urn contains 6 balls numbered from 0 to 5. With replacement, we draw 3 balls. What is the probability that the drawn numbers, in the ordered drawn, are the digits of

a. a 3-digit number (A event);

b. a 3-digit even number (B event);

c. a 3-digit odd number (C event).

Solution to Problem 7

Digits: $0, 1, 2, 3, 4, 5$. This time, with replacement, we draw 3 balls.
$\#\Omega = V_6^{3(i)} = 6^3 = 216$.
$\#A = 5 \cdot 6^2$; $\#B = \#C = 5 \cdot 6 \cdot 3$.
Thus, the required probabilities are:

$$\mathbb{P}(A) = \frac{\#A}{\#\Omega} = \frac{5 \cdot 6^2}{6^3} = \frac{5}{6} = 0.8333,$$

$$\mathbb{P}(B) = \mathbb{P}(C) = \frac{\#B}{\#\Omega} = \frac{5 \cdot 6 \cdot 3}{6^3} = \frac{5}{12} = 0.4167.$$

End of Seminar 1