Design of Machines and Structures, Vol. 13, No. 2 (2023), pp. 31–44.
https://doi.org/10.32972/dms.2023.015

NEW METHOD FOR THE GEOMETRIC DESIGN OF THE RUNNER WHEEL OF THE BÁNKI TURBINE

SÁNDOR HAJDÚ

University of Miskolc, Institute of Energy Engineering and Chemical Machinery
3515, Miskolc-Egyetemváros
hajdusor@gmail.com
https://orcid.org/0009-0007-4291-6673

Abstract: This article is a continuation of a previous publication by Author. Paper (Hajdú, 2022) analyses the velocity relationships in the flow stream through the Bánki turbine. By exploring the velocity relationships, it is possible to determine the flow losses in the turbine runner and the runner efficiency of the runner. By implementing the theoretical results in practice, a new design procedure for the geometry of the runner of the Bánki turbine is developed.

Keywords: *streamline, methodology, runner efficiency, Bánki-turbine*

1. INTRODUCTION

The Hungarian literature on Bánki's turbine has long lacked further development of the theory, which has stagnated since Bánki's time. In order to move forward, it was necessary to turn the research work in a completely new direction, breaking away from the available precedents in the Bánki turbine literature. Important result of this research work, beyond the results summarised in (Hajdú, 2022), is the new design procedure based on the determination of the runner efficiency of the Bánki turbine runner. The interpretation of the indices assigned to the velocity and the direction angle of the velocity on the radius circle is the same as it was given in the above-mentioned literature. Likewise, the interpretation of the x, y orthogonal coordinate system and the r, ϕ polar coordinate system are also the same.

2. THE BANKI TURBINE IN THE PARTIAL OPENING CONDITION

Quantities for partial openings are indicated by the subscript i. For full closure of the waterway $i = 0$, for full opening $i = 1$. The regulating tongue is rotated by an angle δ_1 from its position of full closure to its position of full opening (Figure 1.). In the case of partial opening, the angle of rotation of the control tongue is less than δ_1, as expressed by the following relation: $\delta_i = i\delta_1; \ldots 0 \leq i \leq 1$.

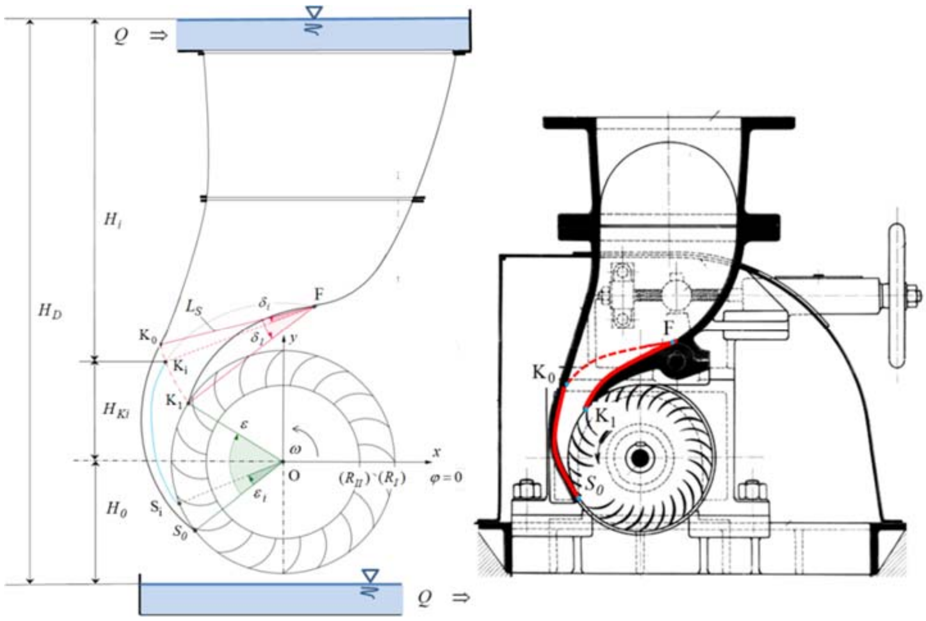

Figure 1. *Schematic of the Bánki turbine in the partial opening condition*

The flow at the runner inlet in the partial opening condition is assumed to be

- the direction angle of the flow on the section S_iS_0 (Figure 1.) of the runner circumference is equal to the direction angle α_0,
- the central angle associated with the section S_iS_0 varies in proportion to the opening parameter i: $\varepsilon_i = i\varepsilon$; $0 \leq i \leq 1$

The variation of the $H_i < H$ head as a function of the opening determines the $c_{1,i} < c_1$ flow velocity of the fluid on the runner.

$$c_{1,i} = \sqrt{2gH_i} = c_1\sqrt{H_i/H} \tag{1}$$

The flow velocity at the inlet in the direction perpendicular to the inlet surface:

$$c_{1m,i} = c_{1,i} \sin \alpha_0 = \sin \alpha_0 \sqrt{2gH_i} = c_1 \sin \alpha_0 \sqrt{H_i/H} \qquad (2)$$

The volume flow rate through the runner varying as a function of the opening parameter:

$$Q_i = R_I \varepsilon_i L c_{1m,i} = R_I i \varepsilon L c_1 \sin \alpha_0 \sqrt{H_i/H} = iQ\sqrt{H_i/H}\ ; \quad 0 \le i \le 1 \qquad (3)$$

3. THE EFFICIENCY SHELL DIAGRAM OF THE BÁNKI TURBINE

In paper (Hajdú, 2022), equations (16) and (17) interpret the runner efficiency for the partial opening condition as the difference between the theoretical runner power and the contraction, collision, and friction performance losses in the runner in the partial opening condition. By combining the above equations (16) and (17), a formula is obtained to calculate the runner efficiency for the partial opening condition:

$$\eta_{jk,i} = \frac{P_{jk,i}}{P_H} = \frac{P_{E,i} - P'_{0,i} - P'_{K,i} - P'_{V,i}}{P_H} = C_{2,i}\psi^2 + C_{1,i}\psi \qquad (4)$$

$$C_{2,i} = -\frac{i}{1+\cot^2 \alpha_0}\left(\frac{H_i}{H}\right)^{3/2}\left[2 + \frac{2\cot \beta_1}{\psi_0} + \zeta_{K,i}\left(S_{II}^2 - \frac{1}{\psi_0^2 S_{II}^2}\right) - \frac{\zeta_{V,i}}{\psi_0^2 S_{II} \sin \beta_1}\right] \qquad (5)$$

$$C_{1,i} = \frac{2i}{1+\cot^2 \alpha_0}\left(\frac{H_i}{H}\right)^{3/2}\left[3\cot \beta_1 + \psi_0 - \frac{\zeta_{K,i}}{\psi_0 S_{II}^2} - \frac{\zeta_{V,i}}{\psi_0 S_{II} \sin \beta_1}\right] \qquad (6)$$

The equation (4) obtained for the runner efficiency of the runner interprets a surface in the three-dimensional rectangular coordinate system i, ψ, η and can therefore be used to construct a shell diagram of the Bánki turbine runner efficiency. This shell diagram is nothing more than the image of the surface described by the previous equation, represented by the $\eta_n = const$ level lines. The implicit equation of such a level line is:

$$C_{2,i}(i)\psi^2 + C_{1,i}(i)\psi - \eta_n = 0 \tag{7}$$

The solution of this second-degree algebraic equation (7) for ψ:

$$\psi_{1,2}(i) = \frac{-C_{1,i}(i) \pm \sqrt{C_{1,i}^2 + 4C_{2,i}\eta_n}}{2C_{1,i}(i)} \tag{8}$$

which provides the upper and lower sections of the level line in question in the orthogonal coordinate system (Figure 2).

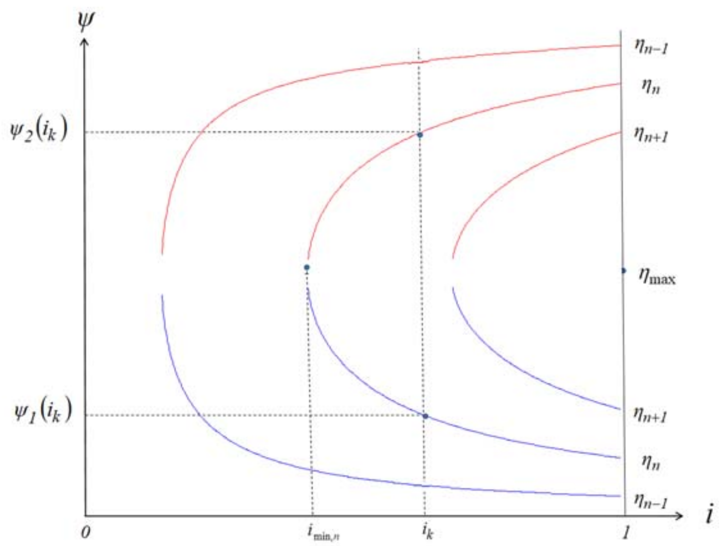

Figure 2. *Level lines of the runner efficiency surface of the runner on the i, ψ coordinate plane*

The interpretational range of the efficiency curve in the i, ψ coordinate plane is $i_{min,n} \leq i \leq 1$; and this curve has a peak at the abscissa $i = i_{min,n}$ (Figure 2.). At the peak of the curve, the tangent to the curve is vertical, and the two curve segments are located to the right of this point. The equation $C_{1,i}^2 + 4C_{2,i}\eta_n = 0$ is true for the abscissa at the peak of the curve. The abscissa value can be calculated from the above equation by a series of trial-and-error operations.

4. DESIGN OF THE BÁNKI TURBINE

The design task is to determine the main geometric dimensions of the Bánki turbine that can optimally utilize the $P_D = \rho g Q H_D$ power coming from the river at a given location for a given pair of values Q (flow rate to the turbine) and H_D (difference in level of the dam).

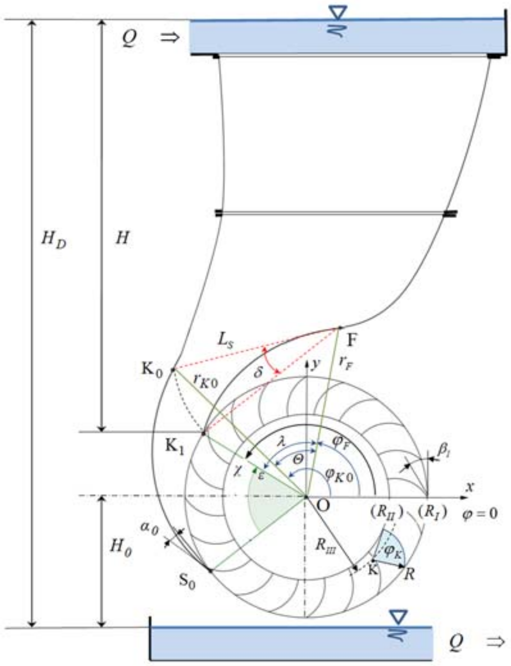

Figure 3. *The angles and radii determining the installation of the Bánki turbine and the runner bladed space and the logarithmic spiral guideway walls*

The initial design data are:
(a) the volumetric flow rate of the water flow through the turbine Q (m^3/s);
(b) the difference in dam level at the location H_D (m);
(c) the ratio of the runner radii $R_{II}/R_I = S_{II}$;
(d) the relative width of the runner L/R_I;
(e) the relative height of the centre of the runner above the sub-water level H_0/R_I;
(f) the angle of the centre of the guide tongue λ;
(g) the number of runner blades N;

(h) the angle of entry of the blades β_1;
(i) the angle of direction of the absolute flow defined by the guide channel α_0;
(j) the centre angle associated with the full opening condition ε;
(k) the angular coordinate value defining the position of the end point of the water flow control tongue in the r, φcoordinate system at full opening χ.
For notations, see the notations in Figure 3.; L is the length of the runner blade measured perpendicular to the blade.
The Bánki turbine shall be designed a) for an impact-free flow entering the runner (i.e., the operating condition $\psi_0 = cot\,\alpha_0 - cot\,\beta_1$ (Hajdú, 2022) is the design operating condition) and b) for the full opening condition of the turbine.
Using the value of the initial data (a) to (k), the main data of the Bánki turbine can be calculated using the equations listed below.
The power coming from the river:

$$P_D = \rho\, gQH_D \tag{9}$$

The size R_I and the value of the available head Hcan be determined iteratively using the following two equations. The size of the inlet surface $R_I\varepsilon\, L$ at the runner peripheral face affected by the inlet is given by the ratio of the volumetric flow rate Q through the runner at full opening divided by the radial (meridional) inlet velocity $c_{1m} = \sqrt{2gH}\, sin\,\alpha_0$, and hence:

$$R_{I,i+1} = \sqrt{\frac{Q}{\varepsilon\ L/R_I\sqrt{2gH_i}\, sin\,\alpha_0}} \tag{10}$$

From equation (1) in (Hajdú, 2022):

$$H_{i+1} = H_D - R_{I,i+1}(H_0/R_I + sin\,\chi) \tag{11}$$

At the beginning of the iteration $(i = 0)$ $H_0 = H_D$.

The inlet is impact-free if the runner speed in RPM is:

$$n_0 = \frac{30\sqrt{2gH}\, sin\,\alpha_0}{R_I\pi}\psi_0 = \frac{30\sqrt{2gH}}{R_I\pi}(cos\,\alpha_0 - cot\,\beta_1) \tag{12}$$

The radius of the blade arc can be calculated from Figure 3 in (Hajdú, 2022):

$$R = \frac{R_I(1 - S_{II}^2)}{2\cos\beta_1} \tag{13}$$

Centre angle of the blade arc is:

$$\varphi_k = \pi - \beta_1 - \arctan\left(\frac{1 - S_{II}^2}{1 + S_{II}^2}\tan\beta_1\right) - \arctan\left(\frac{S_{II}R_I}{R}\right) \tag{14}$$

Length of the blade arc is: $L_k = R\phi_k$. Runner width is: $L = R_I(L/R_I)$.
The relative flow in the blade channels can be considered as a rectangular cross-sectional pipe flow. According to (Hajdú, 2022), the length of the pipe is: $L_{cs} = L_k N\frac{\varepsilon}{\pi}$ and the hydraulic diameter of the pipe is: $D_H = 4\frac{L\pi R_I}{LN+2\pi R_I}$. The velocity of fluid flow in the pipe is: $\bar{w} = \sqrt{\frac{2gH}{S_{II}\sin\beta_1(1+\cot^2\alpha_0)}}$. The Reynolds number of the pipe flow is: $Re = \frac{\bar{w}D_H}{\upsilon}$, $\upsilon = 10^{-6}\ m^2/s$.
Knowing the Reynolds number, the pipe friction coefficient λ_{cs} can be determined by a numerical approximation method taking into account that $\kappa = 0.407$. Knowing the pipe friction coefficient λ_{cs}, the friction loss factor is: $\zeta_V = \lambda_{cs}\frac{L_{cs}}{D_H}$. The contraction loss factor according to (Hajdú, 2022) is: $\zeta_K = \left(\frac{\varepsilon\sin(\varepsilon/2)}{2(1-\cos(\varepsilon/2))} - 1\right)^2$.
The runner efficiency of the Bánki turbine runner in the operating condition ψ_0 is:

$$\eta_{jk}(\psi_0) = 1 - \frac{1 + (\psi_0 - \cot\beta_1)^2}{1 + \cot^2\alpha_0} - \frac{\zeta_K(1 + S_{II}^4\psi_0^2)}{S_{II}^2(1 + \cot^2\alpha_0)} - \frac{\zeta_V}{S_{II}\sin\beta_1(1 + \cot^2\alpha_0)} \tag{15}$$

The resulting efficiency in the operating condition ψ_0 is interpreted as the ratio of the runner power to the power coming from the river:

$$\eta_{resulting}(\psi_0) = \frac{P_{jk}(\psi_0)}{P_D} = \eta_{jk}(\psi_0)\frac{\rho\, gQH}{\rho\, gQH_D} = \eta_{jk}(\psi_0)\frac{H}{H_D} \tag{16}$$

The height difference between the centre of the runner and the sub-water level (Figure 3.) is: $H_0 = R_I(H_0/R_I)$. The radius of the circle delimiting the inside of the runner's bladed space is: $R_{II} = S_{II}R_I$. The radius of a circle concentric with the centre of the runner and representing the geometric location of the centres of the blade arcs

(Figure 3.) is: $R_{III} = \sqrt{R_{II}^2 + R^2}$. The angular coordinate φ_F (Figure 3.) of the point of rotation F of the control tongue (Figure 3.) in the coordinate system r, φ is:$\phi_F = \chi - \lambda$. The radial coordinate r_F of the rotation point F of the regulating tongue (Figure 3.) in the coordinate system r, φ is: $r_F = R_I e^{\lambda \tan \alpha_0}$.
Length of the regulating tongue (Figure 3.) is:

$$L_S = R_I \sqrt{1 + e^{2\lambda \tan \alpha_0} - 2e^{\lambda \tan \alpha_0} \cos \lambda} \tag{17}$$

The angle of the absolute flow leaving the runner is α_4 (as indicated in (Hajdú, 2022)). The following equations are obtained by taking into account equation (7) of (Hajdú, 2022) and equation $\beta_4 = \pi - \beta_1$:

$$\cot \alpha_4 = \psi + \cot \beta_4 = \psi - \cot \beta_1 = (\cot \alpha_1 - \cot \beta_1) - \cot \beta_1 = \cot \alpha_1 - 2 \cot \beta_1 \tag{18}$$

In the impact-free entry mode ($\psi = \psi_0$) $\alpha_1 = \alpha_0$ is valid, hence, according to the (18), $\cot \alpha_4 = \cot \alpha_0 - 2 \cot \beta_1$ is satisfied. This implies that the choice of the value pair $\alpha_0; \beta_1$ in the impact-free entry mode determines the direction angle α_4 of the absolute flow exit, and that the mode characteristic number ψ_p associated with the radial exit is:

$$\psi_p = \cot \beta_1 \quad (\alpha_4 = \pi/2, \qquad \cot \alpha_4 = 0) \tag{19}$$

If the choice of the value pair $\alpha_0; \beta_1$ satisfies the condition $\cot \alpha_0 = 2 \cot \beta_1$, then, according to the (19)

$$\psi_p = \cot \beta_1 = \psi_0 = \cot \alpha_0 - \cot \beta_1 \tag{20}$$

is satisfied, i.e., the radial exit and impact-free entry occur in the same operating condition. It also follows from the above that, if $\alpha_0 = \beta_1/2$, then $\alpha_4 = \mathrm{acot}(\cot \alpha_0 - 2 \cot \beta_1) < \pi/2$ is satisfied and the angle of exit deviates only slightly from $\pi/2$.
Baseline data chosen for the design example are:

$$H_D = 3\ [m], Q = 1\ [m^3/s], L/R_I = 4, R_{II}/R_I = \frac{2}{3}, \frac{H_0}{R_I} = 1.15, N = 20, \lambda = 70°$$

For the design, we choose $\alpha_0;\beta_1$ value pairs, where, based on (20), one β_1 value is assigned three different α_0 values for which the following equations are satisfied:

$$\alpha_0 = \text{acot}(2\cot\beta_1) \rightarrow \alpha_4 = \text{acot}(\cot\alpha_0 - 2\cot\beta_1) = \frac{\pi}{2}$$

$$\alpha_0 < \text{acot}(2\cot\beta_1) \rightarrow \alpha_4 = \text{acot}(\cot\alpha_0 - 2\cot\beta_1) < \frac{\pi}{2} \leftarrow \alpha_0 = \frac{\beta_1}{2}$$

$$\alpha_0 > \text{acot}(2\cot\beta_1) \rightarrow \alpha_4 = \text{acot}(\cot\alpha_0 - 2\cot\beta_1) > \pi/2 \leftarrow \alpha_0 = 1{,}1\text{acot}(2\cot\beta_1)$$

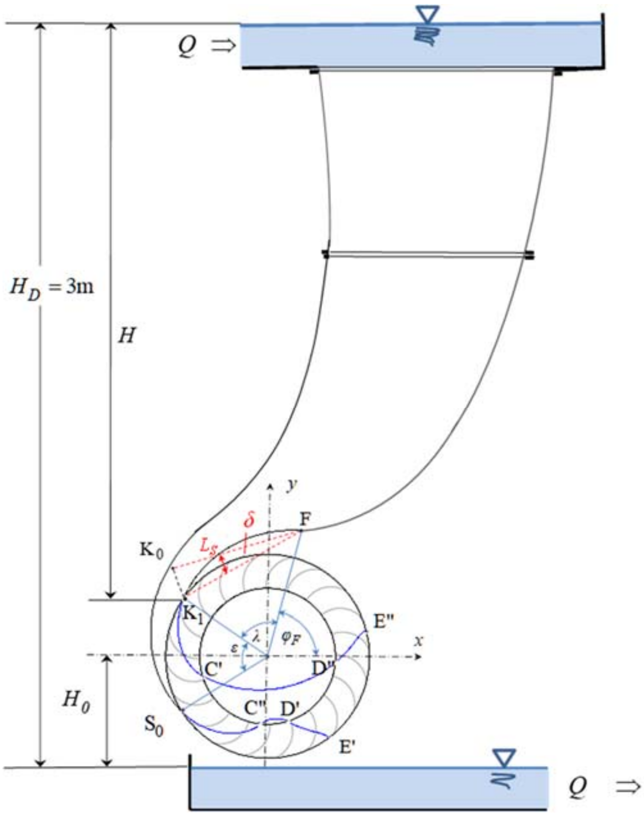

Figure 4. *Scale drawing of the Bánki turbine*

Given the data of the Bánki turbine variants using the different $\alpha_0;\beta_1$ value pairs, a decision can be made on the variant to be implemented. The results of the calculations are easy to review when the data are tabulated. The results of a sample calculation are shown in Table 1. In the calculation example, the data in Table 1 are used to select the variant „c" in the case of $\beta_1 = 20°$ for the Bánki turbine to be implemented in order to achieve a high resulting efficiency, and the Bánki turbine is thus designed. Based on the data determined in the design, a scale plan of the Bánki turbine (Figure 4) and a shell diagram of the runner efficiency of the Bánki turbine runner (Figure 5) can be drawn.

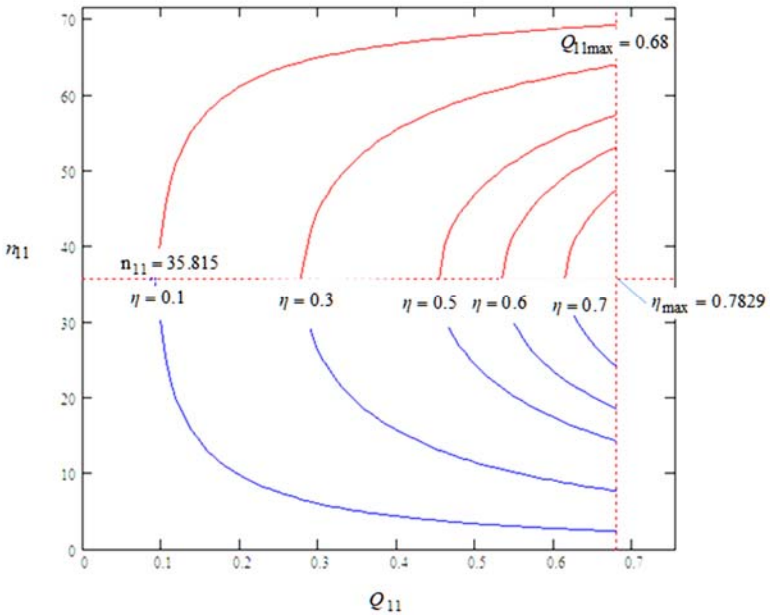

Figure 5. *Shell diagram of the runner efficiency of the Bánki turbine runner*

Table 1.
Calculated results

	a	b	c	a	b	c
β_1°	15			20		
α_0°	7.5	7.631	8.394	10	10.314	11.346
ε °	45	47	55	56	59	68
χ°	147	147	150	151	150	145
$\psi_t = \psi_0$	3.8637	3.7321	3.0450	2.9238	2.7475	2.2364

$\eta_E(\psi_0)$	0.9827	0.9824	0.9686	0.9689	0.9679	0.9512
$\alpha_4(\psi_0)°$	82.5000	90.0000	124.4911	80.0000	90.0000	117.0694
$\varphi_C°$	78.9424	76.8471	65.9123	62.4776	59.6713	51.5377
$\alpha_C°$	30.2141	31.0851	36.4613	37.5799	39.3152	45.1733
$\varphi_D°$	139.3705	139.0174	138.8349	137.6374	138.3016	141.8843
$(\varphi_D - \varphi_C)°$	60.4281	62.1703	72.9226	75.1598	78.6303	90.3467
$\varphi_E°$	183.4135	180.9651	169.8478	168.2273	166.0852	161.5342
$\varphi_{K2}°$	15.4135	14.9651	14.8478	15.2273	15.0852	14.5342
R_I m	0.6291	0.6075	0.5249	0.4719	0.4514	0.3986
H m	1.9339	1.9705	2.1339	2.2286	2.2551	2.3129
R_{II} m	0.4194	0.4050	0.3499	0.3146	0.3010	0.2658
R m	0.1809	0.1747	0.1510	0.1395	0.1334	0.1178
R_{III} m	0.4568	0.4411	0.3811	0.3441	0.3292	0.2907
$\varphi_K°$	92.4497	92.4497	92.4497	85.9439	85.9439	85.9439
L_k m	0.2919	0.2819	0.2436	0.2092	0.2002	0.1768
L m	2.5165	2.4299	2.0996	1.8875	1.8057	1.5946
$\varphi_F°$	77.0000	77.0000	80.0000	81.0000	80.0000	75.0000
r_F m	0.7389	0.7155	0.6286	0.5853	0.5638	0.5094
L_S m	0.7898	0.7640	0.6671	0.6135	0.5896	0.5287
$\Theta°$	66.3442	66.1137	65.0009	64.0709	63.4416	61.6672
$\varphi_{K0}°$	143.3442	143.1137	145.0009	145.0709	143.4416	136.6672
r_{K0} m	0.7035	0.6842	0.6126	0.5710	0.5559	0.5208
$\delta°$	6.2180	6.6287	8.6573	10.5396	11.6029	15.1030
L_{cs} m	1.4596	1.4721	1.4885	1.3019	1.3122	1.3356

D_H m	0.3665	0.3539	0.3058	0.2749	0.2630	0.2322
$\overline{w}$ m/s	1.9356	1.9877	2.2739	2.4046	2.4941	2.7753
Re	709393	703439	695337	661047	655917	644527
λ_{cs}	0.0152	0.0152	0.0152	0.0154	0.0154	0.0154
ζ_V	0.0604	0.0632	0.0741	0.0728	0.0768	0.0888
ζ_K	0.9491	0.9446	0.9244	0.9217	0.9132	0.8854
$\eta_{jk}(\psi_0)$	0.8330	0.8353	0.8340	0.7912	0.7931	0.7829
$P_{jk}(\psi_0)$ kW	15.8036	16.1478	17.4582	17.2965	17.5450	17.7626
n_0 f/p	47.1520	48.4379	52.3227	67.9381	69.2174	70.9942
$\eta_{resulting}(\psi_0)$	0.5370	0.5487	0.5932	0.5877	0.5962	0.6036
	a	**b**	**c**	**a**	**b**	**c**
β_1°	25			30		
α_0°	12.5	13.124	12.5	13.124	12.5	13.124
ε °	65	70	65	70	65	70
χ°	148	144	148	144	148	144
$\psi_t = \psi_0$	2.3662	2.1445	2.3662	2.1445	2.3662	2.1445
$\eta_E(\psi_0)$	0.9509	0.9484	0.9509	0.9484	0.9509	0.9484
$\alpha_4(\psi_0)$°	77.5000	90.0000	77.5000	90.0000	77.5000	90.0000
φ_C°	52.1815	48.6532	52.1815	48.6532	52.1815	48.6532
α_C°	43.5580	46.3751	43.5580	46.3751	43.5580	46.3751
φ_D°	139.2975	141.4034	139.2975	141.4034	139.2975	141.4034
$(\varphi_D - \varphi_C)$°	87.1160	92.7503	87.1160	92.7503	87.1160	92.7503
φ_E°	162.4345	161.0120	162.4345	161.0120	162.4345	161.0120

φ_{K2} °	15.4345	15.0120	15.4345	15.0120	15.4345	15.0120
R_I m	0.3872	0.3636	0.3872	0.3636	0.3872	0.3636
H m	2.3495	2.3682	2.3495	2.3682	2.3495	2.3682
R_{II} m	0.2581	0.2424	0.2581	0.2424	0.2581	0.2424
R m	0.1187	0.1114	0.1187	0.1114	0.1187	0.1114
R_{III} m	0.2841	0.2668	0.2841	0.2668	0.2841	0.2668
φ_K °	79.5223	79.5223	79.5223	79.5223	79.5223	79.5223
L_k m	0.1647	0.1547	0.1647	0.1547	0.1647	0.1547
L m	1.5489	1.4543	1.5489	1.4543	1.5489	1.4543
φ_F °	78.0000	74.0000	78.0000	74.0000	78.0000	74.0000
r_F m	0.5077	0.4834	0.5077	0.4834	0.5077	0.4834
L_S m	0.5227	0.4956	0.5227	0.4956	0.5227	0.4956
Θ°	61.4249	60.2348	61.4249	60.2348	61.4249	60.2348
φ_{K0} °	139.4249	134.2348	139.4249	134.2348	139.4249	134.2348
r_{K0} m	0.5147	0.5030	0.5147	0.5030	0.5147	0.5030
δ°	15.8289	18.2592	15.8289	18.2592	15.8289	18.2592
L_{cs} m	1.1896	1.2030	1.1896	1.2030	1.1896	1.2030
D_H m	0.2256	0.2118	0.2256	0.2118	0.2256	0.2118
$\overline{w}$ m/s	2.7685	2.9159	2.7685	2.9159	2.7685	2.9159
Re	624521	617625	624521	617625	624521	617625
λ_{cs}	0.0155	0.0156	0.0155	0.0156	0.0155	0.0156
ζ_V	0.0819	0.0883	0.0819	0.0883	0.0819	0.0883
ζ_K	0.8951	0.8788	0.8951	0.8788	0.8951	0.8788
$\eta_{jk}(\psi_0)$	0.7386	0.7377	0.7386	0.7377	0.7386	0.7377

$P_{jk}(\psi_0)$ kW	17.0227	17.1388	17.0227	17.1388	17.0227	17.1388
n_0 f/p	85.7510	87.1758	85.7510	87.1758	85.7510	87.1758
$\eta_{resulting}(\psi_0)$	0.5784	0.5824	0.5784	0.5824	0.5784	0.5824

5. SUMMARY

Under the conditions of a circular curve of the runner blade (the tangent to the blade is radial at the inner boundary of the bladed space) and congruent relative flow, the geometry of the runner of the Bánki turbine can be designed and a scale drawing of the turbine installation can be prepared by applying the relationships summarized in the paper. Furthermore, the expected operating data of the designed turbine can be determined, including a shell diagram of the runner efficiency of the runner. Such a calculation has not been reported in the literature so far.

ACKNOWLEDGEMENT

This research was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP-4.2.4.A/2-11/1-2012-0001 'National Excellence Program'.

REFERENCES

Bánki, D. (1920). *Energiaátalakulások folyadékokban* (2.. kiad.). Budapest: Franklin.

Czibere, T. (2009). Az el nem évülő Bánki-turbina létrehozója 150 éve született. *GÉP, 60*(3), 9-15.

Hajdú, S. (2022). Calculation of the flow generated in the runner of a Bánki turbine. *Design of Machines and Structures, 12*(1), 12-25. https://doi.org/10.32972/dms.2022.002

Mockmore, C., & Merryfield, F. (1949). The Banki Water Turbine. *Engineering Station Bulletin Series, 25*.