

INVESTIGATION OF THE INVOLUTE SPUR GEARS WITH DIFFERENT PRESSURE ANGLES

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Abstract: This article deals with an investigation of the choice of the profile shift coefficient for gears when the standard pressure angles of the gears are different. The basic idea for the study arose from a real-life problem where a gear in a piece of equipment needed to be replaced. In this article, a method is presented to determine, by approximation, the value of the profile shift coefficient that we have to use on a gear having 20° pressure angle to get similar tooth profile of a gear without profile shift and 21° of pressure angle. In addition, theoretical investigations are also carried out on the topic of gear meshing, using the profile shift coefficient.

Keywords: *gear CAD model, profile shift coefficient, gear meshing*

1. INTRODUCTION

Gear drives cover a very large area of conventional engineering. It is therefore not surprising that their study is still a very important part of research today. For us, such a topic arose during a real industrial task. In this article, we therefore focus on the study of gear meshing and the choice of the profile shift coefficient. The question is given, whether gears with different pressure angles can be coupled, and if so, with what amount of error. The task was to realize a gear connection with a pressure angle of 21° and a gear connection with a pressure angle of 20°. Therefore, the theoretical gear meshing investigation will be one of the important points. In addition, the use of profile shifting will be used to create gears with different pressure angles but nearly identical tooth profiles. It is also a task in the article to determine what profile shift coefficient should be applied to change the tooth profile of a gear with a pressure angle of 20° to match as closely as possible the tooth profile of a gear without profile shift with a pressure angle of 21°. Finally, it is also investigated whether two gears with different pressure angles can be interchanged without changing the module.

2. MATING GEARS WITH DIFFERENT PRESSURE ANGLES AND MODULES

The question may arise whether it is even possible to design gears with different pressure angles. So, the question is, can a gear with a pressure angle α_1 be replaced by a gear with a pressure angle α_2 ? The meshing of two gears can be achieved if the meshing points run along the common inner tangent of the base circles of the two gears. *Figure 1* is attached for reference. From the figure, we can say that for spur gears, the contact is along the line of action. The line of action itself is a tangent line drawn to the base circle.

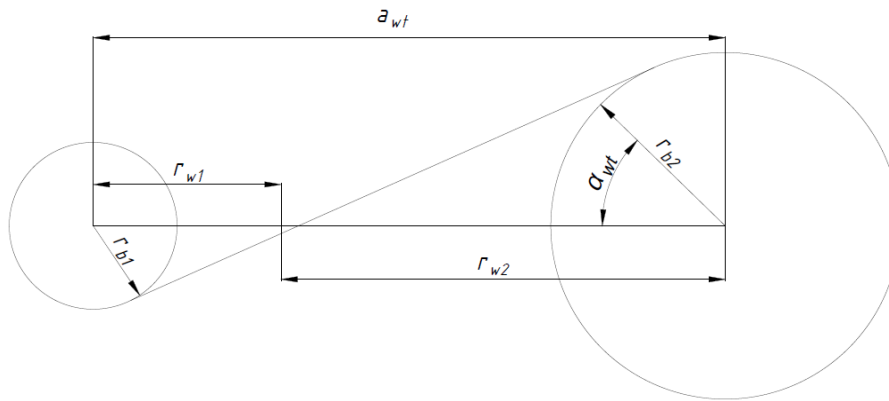


Figure 1. Interpreting the line of action of spur gears

Using the basic assumption that identical involute curves can only be derived from the same base circle (d_b), we can express the diameter of the base circle, which is given by (1):

$$d_b = z \cdot m \cdot \cos \alpha, \quad (1)$$

where z is the number of teeth, m is the modulus and α is the pressure angle. Based on the distinction between the two gears, let number 1 denote the original gear and number 2 the substitute gear. Since the number of teeth cannot vary, the following relationship must be satisfied for the two base diameters to be equal:

$$d_b = z \cdot m_1 \cdot \cos \alpha_1 = z \cdot m_2 \cdot \cos \alpha_2. \quad (2)$$

Rearranging (2) and expressing one of the modules, we obtain the relationship between them:

$$m_2 = m_1 \cdot \frac{\cos \alpha_1}{\cos \alpha_2}. \quad (3)$$

If we start from a different direction, namely from the assumption that the two gears must have the same base pitch (p_b) for a continuous connection, the following relationship is obtained:

$$p_b = m_1 \cdot \pi \cdot \cos \alpha_1 = m_2 \cdot \pi \cdot \cos \alpha_2, \quad (4)$$

which gives the relation (3). As commercially available gear cutting tools are made with a standard set of modules, condition (3) can only be achieved for chipped gears at a significant extra cost. However, if we turn towards additive manufacturing, or consider injection moulded plastic gears, then for these gears the constraints on the module are not present, so condition (3) can be fulfilled without difficulty.

3. MATING GEARS WITH DIFFERENT PRESSURE ANGLES

So, in this study, we are looking for the answer to the question, what kind of mesh is shown between two gears with different pressure angles if the module is unchanged. The starting point is that if the module is the same, the base circle diameters will be different:

$$d_{b1} = z \cdot m_1 \cdot \cos \alpha_1, \quad (5)$$

$$d_{b2} = z \cdot m_1 \cdot \cos \alpha_2. \quad (6)$$

In the case where (5) and (6) hold, the involute curves will also be different because of the difference in the base circles. This divergence is illustrated in *Figure 2*.

Since the centre distance cannot change, the centre of the base circles coincides. If you sweep a straight line on each of the two base circles, you get intersecting involute curves. *Figure 2* shows the tangents of the two base circles and the base circles at the intersection point M. The n_1 and n_2 will be the normal of the involute profiles at point M. By varying the position of point M, i.e., the radius r_y , an infinite number of intersections are possible, whose locations are determined by the profile shift coefficient of the two gears.

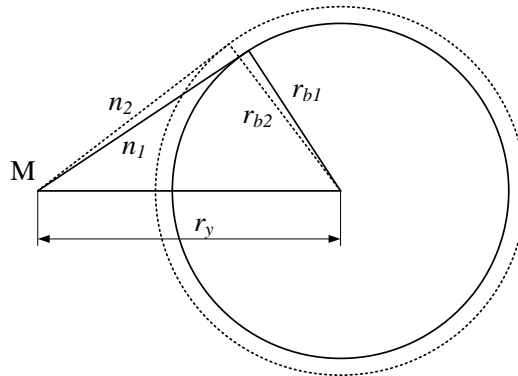


Figure 2. The different base circles and their corresponding tangent lines

Hence, we investigate under what conditions the tooth thickness of the two gears is the same at the radius r_y . To do this, we write down the relationship of the tooth thickness at an arbitrary radius:

$$s_y = 2 \cdot r_y \left[\frac{1}{z} \cdot \left(\frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha \right) + \text{inv} \alpha - \text{inv} \alpha_y \right]. \quad (7)$$

We apply this relationship to the case of the two gears:

$$s_{y1} = 2 \cdot r_y \left[\frac{1}{z} \cdot \left(\frac{\pi}{2} + 2 \cdot x_1 \cdot \tan \alpha_1 \right) + \text{inv} \alpha_1 - \text{inv} \alpha_{y1} \right], \quad (8)$$

$$s_{y2} = 2 \cdot r_y \left[\frac{1}{z} \cdot \left(\frac{\pi}{2} + 2 \cdot x_2 \cdot \tan \alpha_2 \right) + \text{inv} \alpha_2 - \text{inv} \alpha_{y2} \right]. \quad (9)$$

In order to meet the requirement that the two tooth profiles are identical, the tooth thicknesses must be made equal. Thus, the equality between the two tooth thicknesses is given by:

$$\begin{aligned} 2 \cdot x_1 \cdot \tan \alpha_1 + z \cdot (\text{inv} \alpha_1 - \text{inv} \alpha_{y1}) &= \\ = 2 \cdot x_2 \cdot \tan \alpha_2 + z \cdot (\text{inv} \alpha_2 - \text{inv} \alpha_{y2}). \end{aligned} \quad (10)$$

From this relationship we can express one of the profile shift coefficients:

$$x_2 = \frac{2 \cdot x_1 \cdot \tan \alpha_1 + z \cdot (\text{inv} \alpha_1 - \text{inv} \alpha_2 - \text{inv} \alpha_{y1} + \text{inv} \alpha_{y2})}{2 \cdot \tan \alpha_2}. \quad (11)$$

Since in our study we only apply a profile shift coefficient to one of the gears, the relationship for $x_1 = 0$ is as follows:

$$x_2 = \frac{z \cdot (\text{inv} \alpha_1 - \text{inv} \alpha_2 - \text{inv} \alpha_{y1} + \text{inv} \alpha_{y2})}{2 \cdot \tan \alpha_2}, \quad (12)$$

(11), at a given radius r_y , the tooth thicknesses are identical, but at smaller and larger radii the thicknesses of the two gears differ, so that at smaller radii one tooth is thicker and at larger radii the other tooth is thicker. However, if the value of x_2 is chosen carefully, the difference can be minimised within a given r_y range.

In this way we can create a very similar gear. However, a further problem arises, namely that the base pitch of the two gears is not the same, which follows:

$$p_{b1} = m_1 \cdot \pi \cdot \cos \alpha_1, \quad (13)$$

$$p_{b2} = m_1 \cdot \pi \cdot \cos \alpha_2. \quad (14)$$

However, the consequence of the relation (13) and (14) will be that smooth operation is not ensured. Even if we ensure that one tooth is in alignment, the next tooth will come into contact too soon, or too late. The tooth clearance of the replacement wheel may be too wide or too narrow for the tooth of the corresponding gear.

In summary, it is not theoretically possible to mesh gears with different pressure angles but the same module. In practice, with a clever choice of profile shift

coefficient and clearance, it is conceivable, but a very poor-quality pair of gears will result. The consequences can be uneven running, high noise levels and heavy wear.

4. DETERMINATION OF THE PROFILE SHIFT COEFFICIENT

In the previous point, we examined gears that had the same module (standard) and had different pressure angles. If we waive the requirement that the module have to be standard and identical on both wheels (this is possible in the case of additive manufacturing or injection moulding), the question arises as to whether we can better approximate the two profiles.

To create completely identical profiles, it is necessary that the base circle diameter of the two gears, with different pressure angles, be the same.

Then we look for the profile shift coefficient that gives the same base circle for a 20° pressure angle as in the case of a 21° pressure angle non-profile shifted gear. We are looking for equivalence with the help of an example. The base gear, with a pressure angle of $\alpha_1 = 21^\circ$, tooth number $z = 17$, module $m_1 = 3$ mm, profile shift coefficient $x = 0$, (indicate the base gear with index 1).

The radius of the pitch circle:

$$r_1 = \frac{z}{2} m_1 = 25.5 \text{ mm} \quad (15)$$

Radius of the base circle:

$$r_{b1} = r_1 \cdot \cos(\alpha_1) = 23.806 \text{ mm} \quad (16)$$

Radius of the addendum circle:

$$r_{a1} = \left(\frac{z}{2} + 1 + x_1\right) \cdot m_1 = 28.5 \text{ mm} \quad (17)$$

Tooth thicknesses ($s_{y_{ii}}$) can be calculated by arbitrarily dividing the distance between the base circle and the addendum circle. Knowing the radii, the corresponding profile angle can be determined, and then the tooth thicknesses can be determined. Dividing the range between the base circle and the head circle into $I = 5$ equal parts, we get the values in the following table (*Table 1*). The relations used for the calculation are (18)–(21).

The pitch between the radiuses:

$$\Delta r = \frac{r_{a1} - r_{b1}}{5} \quad (18)$$

Radius:

$$r_{y_i} = r_{b1} + i \cdot \Delta r \quad (19)$$

Profile angle:

$$\alpha_{y1} = \cos^{-1} \left(\frac{r_{b1}}{r_{y_i}} \right) \quad (20)$$

Tooth thickness:

$$s_{y1i} = \left(\frac{s_1}{2r_1} + \tan \alpha_1 - \alpha_1 - \tan \alpha_{y1i} + \alpha_{y1i} \right) \cdot 2r_{yi} \quad (21)$$

Table 1

Tooth thickness values for an arbitrary radius

Radius r_{yi} [mm]	Tooth thickness s_{y1} [mm]
23.806	5.225
24.745	5.072
25.685	4.601
26.623	3.902
27.561	3.003
28.5	1.919

Once we know the tooth thicknesses and base diameter of the base gear, we can find out what kind of replacement gear we can create. We are looking for the module and profile shift factor of the new wheel, and we know the pressure angle $\alpha_2 = 20^\circ$, tooth number $z = 17$.

The module of the new gear will differ from the module of the base gear, based on equation (13) and (14):

$$m_2 = m_1 \cdot \frac{\cos \alpha_1}{\cos \alpha_2} = 2.98 \text{ mm} \quad (22)$$

This ensures that the base pitch of the two wheels will be the same. The pitch radius of the new gear (23):

$$r_2 = \frac{z}{2} m_2 = 25.334 \text{ mm} \quad (23)$$

The base circle radius of the new gear is the same as the base circle radius of the base gear.

$$r_{b2} = r_2 \cdot \cos(\alpha_2) = 23.806 \text{ mm} \quad (24)$$

Since the two base radii are the same, the involute of the two gear profiles is also the same. By changing the profile shift coefficient, the involute profile is moved to a new position, from the point of view of the gear, this means a thicker or thinner tooth. We can find a profile shift coefficient at which the two tooth profiles overlap, the tooth thickness of the two gears will be the same.

Using (12) and realizing that the profile angles will be the same at a given radius for the base and replacement gear, (12) can be simplified.

Equality of profile angles:

$$\alpha_{y1} = \cos^{-1} \left(\frac{r_{b1}}{r_y} \right); \alpha_{y2} = \cos^{-1} \left(\frac{r_{b2}}{r_y} \right), \quad (25)$$

but $r_{b1} = r_{b2}$, so $\alpha_{y1} = \alpha_{y2}$, therefore their involutes are also equal. Knowing all this, the profile shift coefficient of the replacement gear is as follows:

$$x_2 = \frac{z}{2 \cdot \tan \alpha_2} \cdot (\tan \alpha_1 - \alpha_1 - \tan \alpha_2 + \alpha_2) = 0,057 \quad (26)$$

If the tooth thicknesses are calculated for the replacement gear, we get the same values as calculated for the base gear. In other words, the profile of the two gears is the same, their module is different, and their pressure angle is different.

5. FURTHER RESEARCH DIRECTIONS

A direction to take this research further could be to investigate the relationship between the mesh of an elementary and a profile shifted gear compared to the mesh of two elementary gears.

The aim is to create assembly models for this purpose, as has been done previously (Pintér & Sarka, 2021), (Pintér & Sarka, 2022), and in these situations to compare, by means of FEM analysis studies, how the values measured at each contact, such as stress distribution, deformation, surface pressure, magnitude of slip, relate to each other. This would also allow us to infer how our gear contact, currently constructed using a profile shift coefficient, with the elemental gear, and resembles the element-element gear coupling.

6. SUMMARY

In the field of mechanical engineering, gear drives are still very common today. Gear drives can be found in many places and are still in widespread use. For this reason, this article has been written about gear meshing. The study looked at the meshing of gears with different pressure angles, whether they can be mesh without failure, and what the conditions are for this mesh. On the topic of profile shift coefficient, we investigated, based on a real industrial problem, how to find a profile shift coefficient that gives a similar tooth profile when the pressure angle is different. We also investigated the theoretical background of this question, how a gear can be substituted when the pressure angle is different if the module is also unchanged. In addition, we have identified the errors and problems that may arise in this case.

ACKNOWLEDGEMENT

The Authors would also like to say thank dr. József Szente, Titular Professor, for his help in researching the topic.

Supported by the ÚNKP-22-2. New National Excellence Program of the Ministry for Culture and Innovation from the source of the National Research, Development and Innovation Fund.



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