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THE VIKOR ALGORITHM IN MATERIAL DECISION SUPPORT

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Abstract: The paper presents how to deal with frequent decision conflicts between design criteria that arise when selecting complex materials. The results show that the VIKOR model, in this case study, can also be used for extensive exploration of trade-offs and design return points, such as changes in environmental, material performance, and cost characteristics for the decision maker during design. In this paper, we examine the problem-solving algorithm for implementation.

Keywords: *Multi-Criteria Decision Making (MCDM), VIKOR, design theory, design methodology*

1. INTRODUCTION

Nowadays, the use of Multi-Criteria Decision Making (MCDM) in material selection processes has become an intensive research area in product development.

The study presents the application of a multi-aspect decision-making method designed for development needs that supports the work of engineers involved in the design and material selection tasks of hip prostheses. Opricovic [1] was the first to examine VIKOR (Vise Kriterijumska Optimizacija I Kompromisno Resenje), a method published in 1998, which focuses on the ranking of alternatives and their compromise selection for difficult-to-reconcile criteria. The advantages and limitations of the decision support application of the VIKOR method are presented using a design theory case study.

After presenting the results, we draw conclusions and make suggestions for further applications of the methodology *(Figure 1)*.

In order to select the right materials, even in the case of alternatives to the simplest products, it is necessary to simultaneously consider many conflicting criteria when ranking them, which is usually a complex problem solution for decision-makers (DMs) *(Figure 2)*.
The select of material is often limited or often based only on experience intended for practical purposes based on the available material properties, which can result in potential underutilization of materials or a reduction in life cycle.

2. REVIEW OF DECISION SUPPORTING METHODOLOGIES

Decision-making tasks are characterized by the fact that each alternative can have positive and negative sides, and the aspects can be both quantitative and non-quantifiable.

The VIKOR decision support method searches for the solution closest to the ideal solution, which is still feasible, from a set of alternatives with contradictory and non-comparable criteria (e.g., attributes with different measurement units).

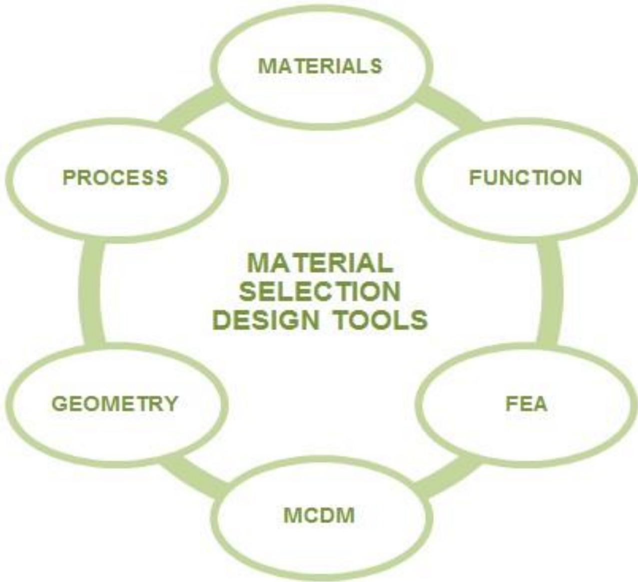

Figure 1. *MCDM as a material design / selection tool* [2]

Its basic concept is based on the definition of positive and negative ideal points in the solution space, and the degree of relative “closeness” to the “ideal” solution. The solution derived in this way is the selection of the alternative that is closest to the positive ideal solution and farthest from the negative ideal solution.

Due to the nature of the task, the model of the interval-based target value VIKOR method may be suitable for selecting the optimal alternative to be determined in our example, since it can be calculated separately whether the point value of each alternative is significantly higher, that is, whether it can actually be considered better from the point of view of the decision maker. Another advantage is that, in the case of criteria that are difficult to reconcile, by focusing on the ranking and selection of the alternatives, a relatively small amount of data enables a sufficient comparison between the alternative.

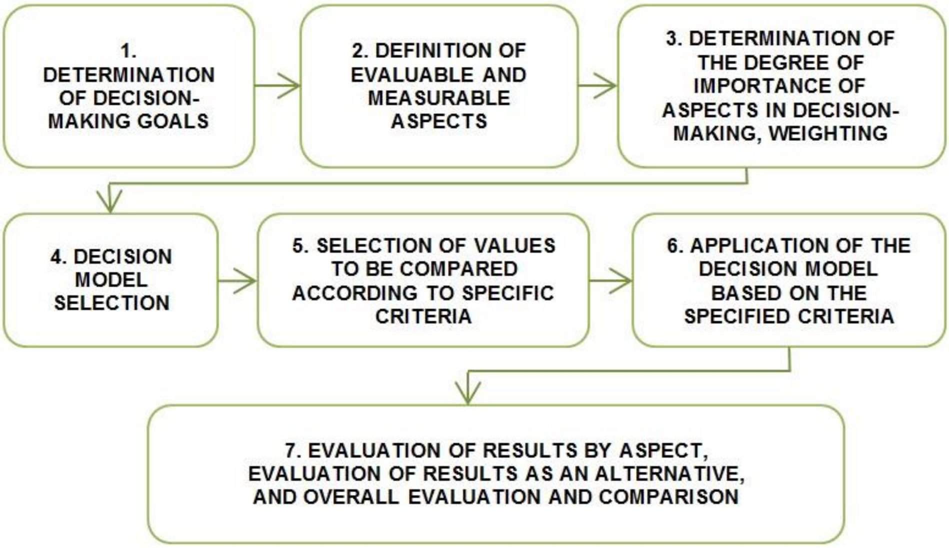

Figure 2. *Flowchart of decision support methodology* [2]

This study presents a method for the selection of hip prosthesis materials, suitable for handling decision conflicts between frequently encountered design criteria, and we illustrate how to be determined the optimal material alternative, with the interval-based target value VIKOR method. It was chosen because it can be calculated separately, whether the score of an alternative is significantly higher, that is, whether it can actually be considered better, in terms of material selection decision. [2]

A further advantage is that, in the case of difficult-to-reconcile criteria, focusing on the ranking and selection of alternatives, a relatively small amount of data allows for a sufficient degree of comparison between the alternatives. The essence of multi-criteria decision support methods is that, in general, conflicting impact criteria must be met at the same time, taking into account the limits of the available data.

The mathematical foundations of the algorithm are presented below, and illustrate the application of the method, whose model is shown in Equations (1) and (2).

$$\begin{array}{cccc} & \boldsymbol{A_1} & \cdots & \boldsymbol{A_n} \\ \boldsymbol{C_1}\,\boldsymbol{w_1} & \boldsymbol{u_i(a_{11})} & \cdots & \boldsymbol{u_1(a_{1n})} \\ & \vdots & \ddots & \vdots \\ \boldsymbol{C_m}\,\boldsymbol{w_m} & \boldsymbol{u_m(a_{m1})} & \cdots & \boldsymbol{u_m(a_{mn})} \\ & \boldsymbol{x_1} & \cdots & \boldsymbol{x_n} \end{array} \tag{1}$$

and

$$y_j = \sum_{i=1}^{n} \boldsymbol{w_i u_j (a_{ij})}\, u_{(x)} \tag{2}$$

where

$\boldsymbol{A_j}$: j[th] alternative;
$\boldsymbol{C_i}$: i[th] aspect;
$\boldsymbol{w_i}$: weight of the i[th] aspect;
$\boldsymbol{a_{ij}}$: value of the j[th] alternative according to the i[th] aspect;
$\boldsymbol{u_i}$: the evaluation (utility) function for the i[th] aspect;
$\boldsymbol{x_j}$: score of the j[th] alternative (place in the ranking).

Below, we present the mathematical foundations of the compromise ranking algorithm of VIKOR1. The $\boldsymbol{x_{ij}}$ elements of the $m \times n$ decision matrix determine the score that can be assigned to the ith alternative and the jth aspect. The decision matrix (3)

$$x = \left(\bar{x}_{ij}\right)_{m,n} \tag{3}$$

Step 1: Determination of the priority values of the aspects (4).

$$\bar{x}_{ij}{}^{+} = \underset{i}{max}\left(\bar{x}_{ij}\right), \qquad \bar{x}_{ij}{}^{-} = \underset{i}{min}\left(\bar{x}_{ij}\right), \tag{4}$$

where, $\bar{x}_{ij}{}^{+}$ aspect j is the best and $\bar{x}_{ij}{}^{-}$is the worst value of aspect j.

Step 2: Calculation of the level of utility and the level of individual dissatisfaction based on *equations (5)* and *(6)*:

$$\boldsymbol{S_i} = \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \boldsymbol{w_j} \frac{(\bar{x}_j{}^{+} - \bar{x}_{ij})}{(\bar{x}_j{}^{+} - \bar{x}_j{}^{-})} \tag{5}$$

$$\boldsymbol{R_i} = \underset{\boldsymbol{j}}{\boldsymbol{max}}\left[\boldsymbol{w_j} \frac{(\bar{x}_j{}^{+} - \bar{x}_{ij})}{(\bar{x}_j{}^{+} - \bar{x}_j{}^{-})}\right] \tag{6}$$

where, $\boldsymbol{w_j}$ is the weighting of the aspects, $\boldsymbol{S_i}$ is the measure of utility and $\boldsymbol{R_i}$ is the measure of individual dissatisfaction.

Step 3: Determination of the value of $\boldsymbol{Q_i}$ based on equation (7):

$$\boldsymbol{Q_i} = v\left(\frac{\boldsymbol{S_i} - \boldsymbol{S^+}}{\boldsymbol{S^-} - \boldsymbol{S^+}}\right) + (\boldsymbol{1} - v)\left(\frac{\boldsymbol{R_i} - \boldsymbol{R^+}}{\boldsymbol{R^-} - \boldsymbol{R^+}}\right) \tag{7}$$

when,

$$\boldsymbol{S^+} = \underset{\boldsymbol{i}}{\boldsymbol{min}}[(\boldsymbol{S_i}),\ \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{m}]$$

$$\boldsymbol{S^-} = \underset{\boldsymbol{i}}{\boldsymbol{max}}[(\boldsymbol{S_i}),\ \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{m}]$$

$$R^{+} = \min_{i} [(R_i),\ i = 1, 2, \dots, m]$$

$$R^{-} = \max_{i} [(R_i),\ i = 1, 2, \dots, m]$$

where, v gives the weighting of the decision-making strategy of 'the majority of criteria' (or 'the maximum group utility'), the value of which varies between 0−1, and the decision-maker determines its value. The decision-maker can also apply the maximization of the criteria's usefulness (v = 1) and the minimum individual dissatisfaction strategy, i.e., the maximization of the individual dissatisfaction values of aspects considered to be of lower importance (v = 0). Another compromise can be given by v 'majority vote' ($v > 0.5$), 'consensus' ($v = 0.5$) or 'veto' ($v < 0.5$). In general, a v value of 0.5 is preferred. In this paper, the value of v is 0.5 (this value gives a result with sufficient accuracy, since most decision-making processes include both decision strategies).

Step 4: The alternatives are ranked based on the $\boldsymbol{Q_i}$, VIKOR index value, according to which the lower the value, the better the ranking of the given alternative.

3. Results of optimization supporting process

In the decision situation examined in our example, the decision maker evaluates a finite number of alternatives based on a finite number of criteria. The alternatives are denoted by $\boldsymbol{A_1} \dots \boldsymbol{A_n}$, and the aspects by $\boldsymbol{C_1}$, $\boldsymbol{C_2}, \boldsymbol{C_3}$ ..., $\boldsymbol{C_m}$.

When evaluating the alternatives, the most basic aspect to be taken into account is the cost aspect, and an important aspect is also the availability of the necessary materials. Another essential aspect is the reliability of the alternatives, as well as their guaranteed lifespan.

As the first step in the choice of material for the hip prosthesis examined in our example, we mapped the hip prosthesis materials found in medical practice *(Table 1)* and their material characteristics. The optimal load absorption of the material selection alternatives can be measured by the structural utilization, i.e. the over- or under-sizing resulting from each design can be specified at this point. The suitability of the basic material of the hip prosthesis can be examined on the basis of several aspects [3, 4]. With the interconnected open pores and large surface area of Porous NiTi alloys are emerged to be one of the promising biomaterials for prosthesis. [5] The hip prosthesis performs such complex functions, where relevant requirements include tolerance, corrosion resistance, compliance with mechanical requirements, flexible compatibility, and weight and cost. Since all material is generate a 'foreign body reaction' when implanted in the body, therefore, biocompatibility is directly related to the corrosion behaviour of the material in a specified solution and the tendency for the alloy to release potential toxic ions [6, 7].

In our case, in the decision situation, denote the material properties n and the number of possible materials m. The evaluation criteria were density (g/cm^3), tensile strength (MPa), modulus of elasticity (GPa), elongation (%), corrosion resistance, wear resistance and ossification efficiency. *(Table 2)* The aim is to find the most suitable raw material based on the selected criteria, or to establish a ranking among the raw materials, which one meets the given expectation (8). Let it be

$$[x_{ij}^{L}, X_{ij}^{U}] \tag{8}$$

the interval for the jth characteristic of the ith material, where i= 1, ..., m, j = 1, ..., n. In order to make a decision, we need a target value with the properties of the ideal material.

Mark each target value T1, T2, ..., Tn. In order to approximate the target values, the corresponding weighting is required (9), which specifies how important each feature is, and therefore the weights associated with each feature w_1, ..., w_n, mark where $w_j \geq 0$, j = 1,..., n and

$$\sum_{j=1}^{n} w_j = 1 \tag{9}$$

When we want to maximize or minimize a criterion, we can select a maximum or minimum of data for a particular characteristic of the target values *(Table 3)*.

Table 1

Material alternatives considered in the comparative process

Materials	
$\boldsymbol{A_1}$	Stainless steel L316 (annealed)
$\boldsymbol{A_2}$	Stainless steel L316 (cold worked)
$\boldsymbol{A_3}$	Co-Cr alloys (wrought Co-Ni-Cr-Mo)
$\boldsymbol{A_4}$	Co-Cr alloys (castable Co-Cr-Mo)
$\boldsymbol{A_5}$	Ti alloys (pure Ti)
$\boldsymbol{A_6}$	Ti alloys (Ti-6A1-4V)
$\boldsymbol{A_7}$	Ti-6A1-7Nb (IMI-367 wrought)
$\boldsymbol{A_8}$	Ti-6A1-7Nb (Protasul-100 hot-forged)
$\boldsymbol{A_9}$	NiTi SMA
$\boldsymbol{A_{10}}$	Porous NiTi SMA

Table 2
Evaluations criteria of materials

Evaluation criteria	
C_1	Density (g/cm^3)
C_2	Tensile strength (MPa)
C_3	Modulus of Elasticity (GPa)
C_5	Elongation (%)
C_6	Corrosion resistance, biocompatibility
C_7	Wear resistance
C_8	Osseointegration

By processing the properties of the alternatives, we ranked the alternatives in an Excel implementation using the mathematical model of the VIKOR method. *Table 4* presents the results of the evaluation process.

Table 3
Aspects taken into account during the process of comparing alternatives and their importance value

Evaluation c	**w_i importance**	**Target value**	**Max**	**Min**
C_1	0.071429	1.3	9.13	4.3
C_2	0.1071429	1240	1240	517
C_3	0.1428571	16	240	15
C_4	0.1071429	54	54	10
C_5	0.1785714	0.955	0.955	0.665
C_6	0.202381	0.955	0.955	0.59
C_7	0.190476	0.955	0.955	0.5

Table 4
S, R and Q scores and rank

Weight of aspects	**0.5**	**Criteria's usefulness**	**0.5**				
Materials	S_i^L	S_i^U	R_i^L	R_i^U	Q_i^L	Q_i^U	**Rank**
A_1	0.852931	0.8529314	1	1	1	1	10
A_2	0.784044	0.7840443	1	1	0.9382489	0.9382489	9
A_3	0.623685	0.6723866	1	1	0.7945014	0.8381578	8
A_4	0.651828	0.7005293	0.9955556	1	0.8001732	0.8633852	7
A_5	0.475071	0.4750706	1	1	0.6612816	0.6612816	5
A_6	0.478275	0.4782752	0.9545455	0.9545455	0.4641542	0.4641542	3
A_7	0.492118	0.501642	1	1	0.6765632	0.6851005	6
A_8	0.453479	0.4804736	0.8863636	1	0.1419267	0.6661249	2
A_9	0.360503	0.360503	1	1	0.5585819	0.5585819	4
A_{10}	0.295151	0.2951514	0.545455	0.9545455	0.3	0.3	1

As a result of the ranking of the VIKOR method, among the alternatives, the tenth alternative A_{10} Porous NiTi SMA (Shape Memory Alloy) was ranked the best, ahead of the A_8 and A_6 titanium alloys, and the first alternative A_1 Stainless steel L316 (annealed) was the one that came to the bottom of the ranking.

4. SUMMARY

By applying a ranking method that supports the selection of different alternatives, based on an individual decision-maker's evaluation criteria system, based on our results, it can be stated that the VIKOR method can be effectively used to support the design processes of hip prostheses.

During the evaluation, it can be established that the disadvantages of the method include the fact that the result only gives a ranking between the alternatives, so there is not enough information available about the magnitude of the difference between the alternatives, so the decision-maker receives a certain ranking, but does not have information about the proportions of the differences. Therefore, it would be advisable to use the AHP (Analytical Hierarchy Process) for the further development of the methodology, which makes this information determinable, thereby increasing the effective support of decision makers.

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