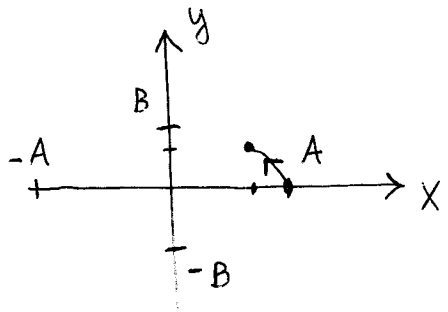
12.)



$x(t) = A\cos\omega t \qquad y(t) = B\sin\omega t$

$t_0 = 0 \qquad t_1 = \frac{\pi}{4\omega} \qquad W = ?$

$$\omega = \frac{2\pi}{T} \qquad W = \int_0^t \vec{F}\cdot\vec{v}\,dt = \int_{\vec{r}_0}^{\vec{r}_1} \vec{F}\cdot d\vec{r}$$

$$\vec{F} = -\nabla V \qquad \frac{d\vec{r}}{dt} = \vec{v}$$

$$t_1 = \frac{\pi\cdot T}{4\cdot 2\pi} = \frac{T}{8} \qquad \cos\omega\frac{\pi}{4\omega} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

A módszer:

$v_x(t) = -A\omega\sin\omega t \qquad v_y(t) = B\omega\cos\omega t$

$F_x(t) = ma_x(t) = -A\omega^2 m\cos\omega t \qquad F_y(t) = -B\omega^2 m\sin\omega t$

$$W = \int_0^{\pi/4\omega} \vec{F}\cdot\vec{v}\,dt = \int_0^{\pi/4\omega} \left(A^2\omega^3 m\sin\omega t\cos\omega t - B^2\omega^3 m\sin\omega t\cos\omega t\right)dt =$$

$$= m\omega^3(A^2-B^2)\int_0^{\pi/4\omega}\sin\omega t\cos\omega t\,dt = \frac{m\omega^2(A^2-B^2)}{2}\left[\sin^2\omega t\right]_0^{\pi/4\omega} =$$

$$= \underline{\underline{\frac{m\omega^2(A^2-B^2)}{4}}} \checkmark$$

B módszer:

$$F_x = -\omega^2 mx \qquad F_y = -\omega^2 my \rightarrow \vec{F} = -\omega^2 m\vec{r} = -\nabla V$$

$$V = \frac{1}{2}\omega^2 m r^2 = \frac{1}{2}m\omega^2(x^2+y^2)$$

$$W = -\Delta V = -(V_2 - V_1) = V_1 - V_2 = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{2} - \frac{B^2}{2}\right) =$$

$$= \underline{\underline{\frac{m\omega^2(A^2-B^2)}{4}}} \checkmark$$