21.)

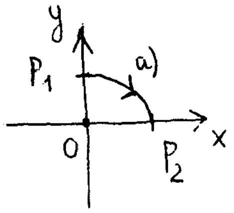
$F_x = ky^2$

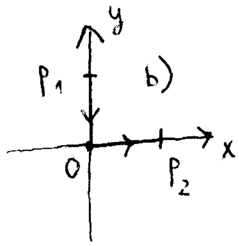
$F_y = kxy$ ($k>0$ áll)

$P_1 = (0, r)$ $P_2 = (r, 0)$

$W_{P_1P_2}$ a) negyed körív

b) P_1O és OP_2





$$\boxed{W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}}$$

a)

$x = r\cos\varphi$ $y = r\sin\varphi$ $\varphi: [\frac{\pi}{2}, 0]$

$F_x = kr^2\sin^2\varphi$ $F_y = kr^2\cos\varphi\sin\varphi$

$dx = \frac{dx}{d\varphi}d\varphi = -r\sin\varphi\, d\varphi$

$\vec{F}\cdot d\vec{r} = F_x dx + F_y dy$

$dy = \frac{dy}{d\varphi}d\varphi = r\cos\varphi\, d\varphi$

$$W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{r} = \int_{\pi/2}^{0} \left(kr^2\sin^2\varphi(-r\sin\varphi) + kr^2\cos\varphi\sin\varphi\, r\cos\varphi \right) d\varphi =$$

$$= \int_{\pi/2}^{0} kr^3(\cos^2\varphi\sin\varphi - \sin^3\varphi)d\varphi = kr^3\int_{\pi/2}^{0} \sin\varphi(\cos^2\varphi - \sin^2\varphi)\,d\varphi = \ldots$$

$$= kr^3\int_{\pi/2}^{0} (2\cos^2\varphi\sin\varphi - \sin\varphi)d\varphi = kr^3\left[\cos\varphi - \frac{2}{3}\cos^3\varphi\right]_{\pi/2}^{0} = \ldots,$$

b)

$P_1O: d\vec{r} = dy\,\hat{y}$ $OP_2: d\vec{r} = dx\,\hat{x}$

$$W = \int_{r}^{0} F_y(x=0)\,dy + \int_{0}^{r} F_x(y=0)\,dx = \ldots$$