SPECIAL THEORY OF RELATIVITY

Relativity:

One principal subject of physics is **relativity**, the field of study that examines the relationship between **space** and **time,** measures events (things that happen), where and when they happen, and by how much any two events are separated in space and in time. In addition, relativity has to do with transforming such measurements (and also measurements of energy and momentum) between reference frames that move relative to each other.

Special relativity (also known as the special theory of relativity) is generally accepted and experimentally confirmed. Einstein's special theory of relativity is based on two postulates:

1. **The Relativity Postulate:** The laws of physics are invariant (identical) in all inertial frames of reference (i.e. non-accelerating frames of reference). This means that the laws of physics are the same for observers in all inertial reference frames (so they are actually not "relative"); and

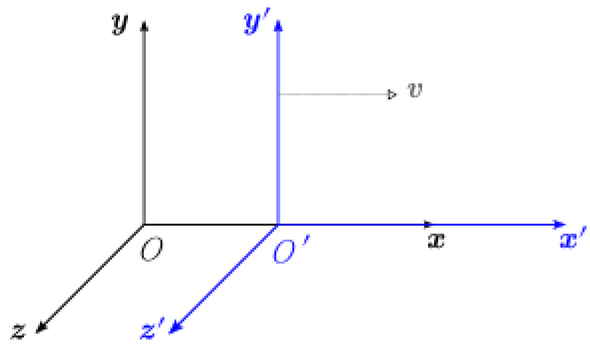
2. **The Speed of Light Postulate:** The speed of light in vacuum has the same value '*c*' in all directions and in all inertial reference frames. Meaning that the speed of light in a vacuum is the same for all observers, regardless of the motion of the light source or observer.

Three space coordinates and one-time coordinate specify an event. One task of special relativity is to relate these coordinates as assigned by two observers who are in uniform motion with respect to each other. If two observers are in relative motion, they generally will not agree as to whether two events are simultaneous.

Special relativity was originally proposed by Albert Einstein in a paper published in 1905. The adjective *special* means that the theory deals only with **inertial reference frames,** which are frames in which, according to classical physics, Newton's laws are valid, hence the term *relativity* here implies only inertial reference frames.

Lorentz transformations and its inverse:

The Lorentz transformation equations relate the spacetime coordinates (t,x,y,z) and (t',x',y',z') of a single event as seen by observers in two inertial frames, *S* and *S'*, where *S'* is moving relative to *S* with velocity *v* in the positive *x* and *x'* direction with respect to that frame.



The four coordinates are related in the following way:

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

$$t' = \gamma(t - vx/c^2)$$

Where

$$\gamma = \frac{1}{\sqrt{1-(v^2/c^2)}}$$

is known as the Lorentz factor, *c* is the speed of light in vacuum and *v* is the velocity of S' relative to S, parallel to the x- axis.

In the above-mentioned transformation, the y and z coordinates are unaffected but only x and t are changed.

For the inverse Lorentz transformation, the above equations are solved for the unprimed coordinates,

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

The transformations can be applied not only to the x axis but also to the y or z axis or any direction parallel to the motion and perpendicular which are warped by the γ factor. A quantity invariant under Lorentz transformation is known as Lorentz scalar.

Some consequences of the Lorentz Equations

Special theory of relativity has wide range of consequences and these consequences of special relativity can be derived from the Lorentz transformation equations. These have been experimentally verified and includes time dilation, length contraction, relative mass and relative velocity. The equations of Lorentz transformations are used to affirm some of the conclusions reached by arguments based directly on the postulates.

I. Time Dilation:

The time lapse between two events is not invariant from one observer to another but is dependent on the relative speeds of the observer's reference frames.

Let's assume a clock is at rest in the unprimed system S. The location of the clock on two different ticks can be characterized by $\Delta x = 0$. To find the relation between the ticks as measured in both systems, take the equation;

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$$

Where $\Delta x = 0$, becomes:

$$\Delta t' = \gamma \Delta t$$

The above equation shows that the time between the two ticks $\Delta t'$ as seen from the frame of which the clock is moving S' is longer than the time Δt between these ticks as measured in the rest frame of reference S. Time dilation explains many physical phenomena, for example the lifetime of high speed muons is greater than the lifetime of slowly moving muons.

The Twin Paradox is one of the common consequence of time dilation, in which a twin flies off in spaceship near the speed of light and returns to discover the non-travelling sibling has aged much more. The paradox being that at constant velocity we are unable to discern which twin is non- travelling and which twin travels. The paradox can be resolved only by considering accelerations, which won't be done here.

II. Length Contraction:

The dimension (particularly the length) of an object may be smaller to one observer than the other observer measuring the same object. Let's assume a measuring rod at rest and aligned along the x-axis in the unprimed system S. Let the length of the rod in this system be Δx. To measure the length of rod in the system S' in which the rod is moving then the distance x', then the distances to the end points of the rod must be measured simultaneously in the system S'. In other words, the measurement is characterized by $\Delta t' = 0$, if substituted in the equation;

$$\Delta x = \gamma(\Delta x' - v\Delta t')$$

Where $\Delta t' = 0$, then the equation becomes:

$$\Delta x' = \frac{\Delta x}{\gamma}$$

The above equation shows that the length $\Delta x'$ of the rod as measured in the frame in which it is moving S', is shorter than its length Δx in its own rest frame S. Length contraction relates to the measured distances between separated but simultaneous events in a given coordinate system of choice. If these events do not happen in the same coordinate system but are separated by distance, they will not occur in the same spatial distance from each other when seen from another moving coordinate system.

III. Relative mass:

In special relativity, an object that has nonzero rest mass cannot travel at the speed of light. As the object reaches the speed of light, the object's energy and momentum increase without bound and the mass becomes infinite.

The total energy and mass of a body can be defined as:

$$m_{rel} = \frac{E}{c^2}$$

where m_{rel} is relativistic mass,

For a body at rest;

$$m_o = \frac{E_o}{c^2}$$

where m_o is the rest mass.

With the ratio,

$$\frac{m_{rel}}{m_o} = \gamma$$

The relationship between rest mass and relative mass can be defined as:

$$m_{rel} = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where, v is the velocity and c is the speed of light.

IV. Relative Velocity:

According to theory of special relativity, the frame of an object travelling at a velocity has a different clock rate and distance measure, and the direction of motion of the object is altered for two observers and the velocity is changed. This applies to all velocities, but can be significant only at near speed of light. The addition of this velocity is called the addition law or composition law for velocities.

Consider two observers with relative velocity v and let u be the velocity of an object relative to one observer and u' be the velocity relative to the other observer. Then the relativistic velocity addition can be denoted as:

$$\boldsymbol{u} = \frac{\boldsymbol{v} + \boldsymbol{u'}}{\boldsymbol{1} + \frac{\boldsymbol{vu'}}{\boldsymbol{c^2}}}$$

In which the term $\frac{vu'}{c^2}$ becomes very small at low velocities.

If v = 0, u = u'

On the other hand, if v approaches c, u also tends to c but cannot be larger.

<u>Mass-energy equivalence</u>

The word *mass* has two meanings in special relativity:

a) **Rest mass** or **invariant mass** is an invariant quantity which is the same for all observers in all reference frames, while
b) **Relativistic mass** is dependent on the velocity of the observer.

According to the concept of mass–energy equivalence, the rest mass and relativistic mass are equivalent to the **rest energy** and **total energy** of the body, respectively.

<u>Mass energy or rest energy:</u>

In 1905, Einstein showed that as a consequence of his theory of special relativity, mass can be considered to be another form of energy. Thus, the law of conservation of energy is really the law of conservation of mass–energy. An object's mass m and the equivalent energy E_0 are related by:

$$\boldsymbol{E_0 = m_0 c^2}$$

This energy that is associated with the mass of an object is called **mass energy** or **rest energy.** The second name suggests that E_0 is an energy that the object has even when it is at rest, simply because it has mass.

<u>Total energy:</u>

The equation above gives the mass energy E_0 for any object that is associated with the object's mass m, regardless of whether the object is at rest or moving. If the object is moving, it has additional energy in the form of kinetic energy K. If we assume that the object's potential energy is zero, then its total energy E is the sum of its mass energy and its kinetic energy:

$$E = E_0 + K = m_0 c^2 + K = \boldsymbol{\gamma m_0 c^2}$$

Thus total energy E can also be written briefly as:

$$\boldsymbol{E = mc^2}$$

where we need to keep in mind that m here is the relativistic m_{rel}.

In special relativity, an object that has **nonzero rest mass** cannot travel at the speed of light.

As the speed of the object approaches the speed of light from an observer's point of view, its **relativistic mass** increases and its energy increases and tends to infinity as the velocity tends to the speed of light. It means that when *v* approaches *c*, any further acceleration becomes harder and harder, finally, an infinite force would be necessary to increase the speed above *c*. This is the reason why nothing can be faster than the speed of light from within the observer's frame of reference.

Relativity theory has been verified by many experiments and the operation of some equipment like GPS depend on its validity and precision.