# Solid State Physics and Semiconductors

Drude model

The Drude model of electrical conduction was proposed in 1900 by Drude to explain the transport properties of electrons in materials (especially metals). The model, which is an application of kinetic theory, assumes that the microscopic behaviour of electrons in a solid may be treated **classically** and looks much like a pinball machine, with a sea of constantly jittering electrons bouncing and re-bouncing off heavier, relatively immobile positive ions.

The Drude model considers the metal to be formed of a mass of positively-charged ions from which a number of "free electrons" were detached. These may be thought to have become delocalized when the valence levels of the atom came in contact with the potential of the other atoms. The Drude model neglects any long-range interaction between the electron and the ions or between the electrons. The only possible interaction of a free electron with its environment is via instantaneous collisions.
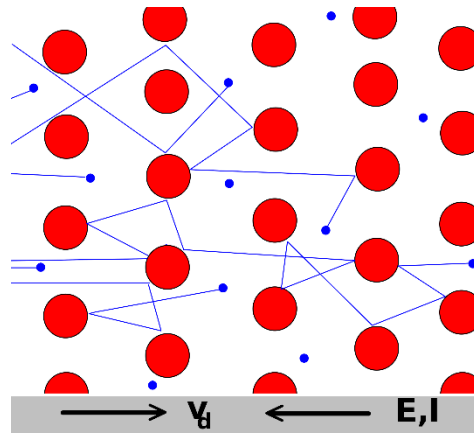
The average time between subsequent collisions of such an electron is denoted by τ, and the nature of the collision does not matter for the calculations and conclusions of the Drude model. Electrons achieve thermal equilibrium by collisions with lattice - they emerge after collision at a random direction with speed appropriate to the temperature of the region where collision happened. The mean free path is denoted by ℓ, thus the average velocity of the electron is: $v = \dfrac{\ell}{\tau}$. Now we would like to derive the $\vec{j} = \sigma \vec{E}$ differential Ohm's law.

If we apply external voltage, an external force will act on the electrons with charge Q=-e :

$$\vec{F} = -e\vec{E}$$

Using Newton's 2. law we obtain the acceleration:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m_e},$$

and the velocity is changed with $\Delta \vec{v} = \vec{a}\tau$. Let us consider it as an average drift velocity caused by the external field:

$$\vec{v}_{drift} = \frac{-e\vec{E}}{m_e}\tau .$$

Now we need to calculate how many electron is „drifting" with this drift velocity. The number density of electrons is the number of all (mobile) electrons divided by the volume of the sample:

$$n = \frac{N}{V} \quad \Rightarrow \quad N = nV$$

Let examine a column-shaped volume in which the electrons pass the A surface of the right. The horizontal size of the column is $s = v_{drift}\tau$, thus the volume of the column is $V = A\, v_{drift}\tau$.
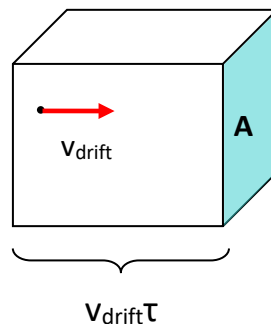
We have $N = nA\, v_{drift}\tau$ electrons in our volume with $Q = -eN = -enA\, v_{drift}\tau$ charge.

The current passing through the surface:

$$I = \frac{Q}{\Delta t} = \frac{-enA\, v_{drift}\tau}{\tau} = -enA\, v_{drift}$$

and using the formula obtained for the drift velocity:

$$I = -enA\, \frac{-eE}{m_e}\tau = \frac{e^2 n\tau A E}{m_e}$$

Thus the current density:

$$\vec{j} = \frac{e^2 n\tau}{m_e}\vec{E}$$

This is Ohm's law. We also obtained a formula for the conductivity:

$$\boxed{\sigma = \frac{e^2 n\tau}{m_e}}$$

It depends on the concentration of electrons and the average time between collisions.
As σ is measurable, if one assume that all valence electrons in a metal take part in the conduction, one can obtain an estimate for τ.

We can easily measure σ and using this we can calculate τ. However, a possible problem arises here. If we take the mean free path ℓ fixed and increase the voltage, the velocity must grow and

therefore τ and σ must decrease. But according to Ohm's law, the proportionality constant cannot depend on U. Let us examine the concrete numbers. The average thermal velocity of the electrons according to classical statistics is $v_T \approx 10^5 \, \frac{m}{s}$. The usual drift velocity is many orders of magnitude smaller, $v_{\text{drift}} \approx 10^{-3} \, \frac{m}{s}$. It means that there is no apparent contradiction in the model.

The calculated ℓ at is approximately $\ell \approx 10^{-9} \, \text{m} = 1\text{nm} \approx 5-10 \, \text{atomic size}$, which is more or less consistent with the assumption of collisions with the fixed ions.

The effect of temperature: If T is growing, v_T is also growing, thus τ and σ is decreasing, which is observed in case of metals.

Problems with the Drude-model:

1. According to the experiments, the resistivity of metals does not go to zero with temperature, but it approaches a constant, which depends on the purity of the material and the perfectness of the crystal structure.
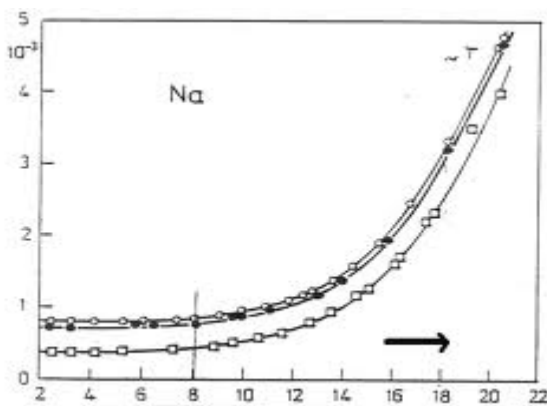
2. The specific heat of the metals can be derived, but not the proper value. The model says the conduction electrons have a large additional contribution to the specific heat of the material, but in the reality this contribution is very small.
3. The model cannot distinguish between metals, semiconductors and insulators, cannot explain the differences.

Therefore we need a more advanced model.

Sommerfeld-modell

Now we change from classical to quantum physics. We assume that the electrons are closed into the metal but inside the metal they still don't interact with the ions of the metal, apart from the collisions. So we treat the electrons using quantum mechanics, but the ions are still classical rigid
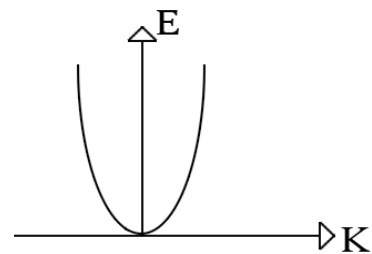
balls. Now our case is very similar to the infinite potential well problem. The solution of the Schrödinger-equation:

$$\psi_k(\vec{r}) = \frac{1}{\sqrt{V}} e^{i\vec{k}\vec{r}}$$,

where the momentum of the electron: $\vec{p} = \hbar\vec{k}$.

Using $E = \dfrac{p^2}{2m_e}$ we obtain that the E(k) function is quadratic:

$$E = \frac{\hbar^2 k^2}{2m_e}$$

It can be derived that the previous formula is valid for the conductivity:

$$\sigma = \frac{e^2 n\tau}{m_e}$$

However, now we have to use the Fermi-Dirac statistic, which take the Pauli principle into consideration. The average thermal velocity of the electrons is much higher than in the Drude model and only slightly depends on the temperature. This explains the specific heat of the metals, i. e. the fact that the conduction electrons have only a small contribution to the specific heat of the material.

However, now the electrons have much higher average speed: $v_T \geq 10^6 \dfrac{m}{s}$. As σ and therefore τ is given by the experiments, the average mean free path must be larger:

$\ell \approx 10^{-8}\,\text{m} = 10\text{nm} \approx 50-100 \text{ atomic size}$ at room temperature, which is strange.

If T is decreasing, σ is growing, but as v_T is roughly constant, we must conclude that ℓ must be even larger, in some cases thousands of the lattice constant. We must consider it as a contradiction, because the atoms in a solid are close to one another, there is not much space between them.
Thus the Drude model's explanation to the temperature-dependence of the conductivity was a false explanation, but the Sommerfeld-model cannot explain the observed facts at all. It also cannot explain that, according to the experiments, the lattice defects increase the resistivity of the metals. Thus we need an even better model.

Electrons in a periodic potential

We assume that the ions are placed in a periodic order in the crystal, thus the electrons move in a periodic potential:

$$V(\vec{r} + \vec{R}) = V(\vec{r})$$
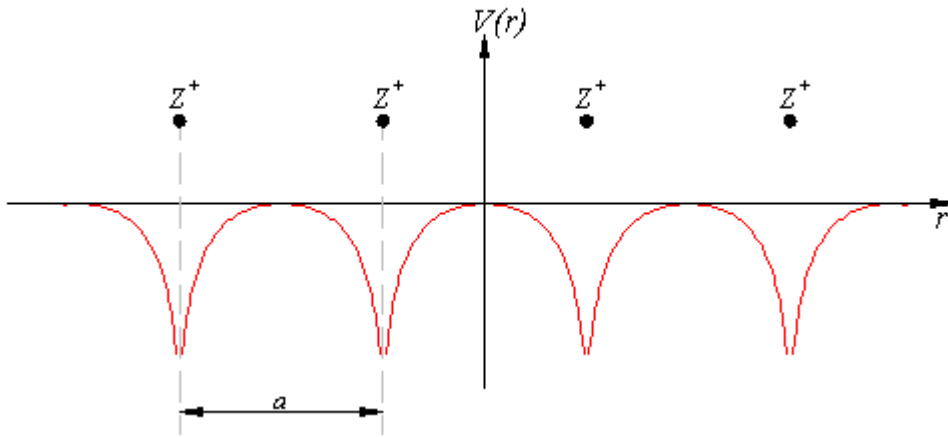
Unfortunately it is very hard to solve the Schrödinger equation in this case. Luckily, there is a theorem which gives us important information without solving the Schrödinger equation.

Bloch-theorem: the stationary wave function of the electrons in a periodic lattice have the following, so called Bloch-wave form:

$$\psi_{nk}(\vec{r}) = e^{i\vec{k}\vec{r}} u_{nk}(\vec{r})$$

where

$u(\vec{r})$ is a real function with the same periodicity as the U potential,

$\vec{k}$ is a vector of real numbers called the (crystal) wave vector.

n is a discrete index, called the **band** index, which is present because there are many different Bloch waves with the same k (each has a different periodic component u). Within a band (i.e., for fixed n), ψ_{nk} varies continuously with k, as does its energy.

So if one multiply a plane wave $e^{i\vec{k}\vec{r}}$ by a lattice-periodic function, one get a Bloch wave.

The quantity $\hbar\vec{k}$ is the so called crystal momentum, very similar to the original momentum; it is better to think of k as a quantum number which describes a Bloch state.

The probability density of a Bloch-state is lattice-periodic: $\left|\psi_{nk}\right|^2 = \left|u_{nk}\right|^2$

The Bloch wavefunctions describe states which are stationary solutions to the Schrödinger equation of a periodic potential. Thus these states persist forever in a perfectly periodic infinite crystal; there will be **no scattering** of electrons in the absence of disorder (which disturbs the perfect periodicity) or boundaries (which destroy the periodicity/infiniteness of the crystal). This is a manifestation of the wave-like nature of electrons; in a periodic array of scatterers a wave can propagate without attenuation because of the coherent constructive *interference* of the scattered waves. Scattering only takes place because of lack of periodicity of the crystal. This observation is in great contrast with the Drude model (where electrons were pictured crashing into virtually every ion) and also with the Sommerfeld approach (where scattering was acknowledged but its cause and mechanism was not well-defined). In addition, this periodic-potential model

- encompasses all possible causes of scattering (oscillation of ions due to temperature, vacancies, impurities, dislocations, grain boundaries, all of which disturb the local periodicity); It is normally a very good approximation to combine their influences using "Matthiessen's Rule":

$$\rho = \rho_T + \rho_{impurities} + \rho_{vacancies} + ...$$

- explains the enormous low-temperature scattering lengths observed in very pure single crystals of metals (often mm or even cm);
- removes the need for the strange explanations as to why the electrons in a metal might not see the ionic cores very much, which was used to justify the Drude and especially the Sommerfeld approaches.

As Bloch-theorem suggest, instead of having discrete energies as in the case of free atoms, the available energy states form bands. The E(k) function is not parabolic any more, but distorted:
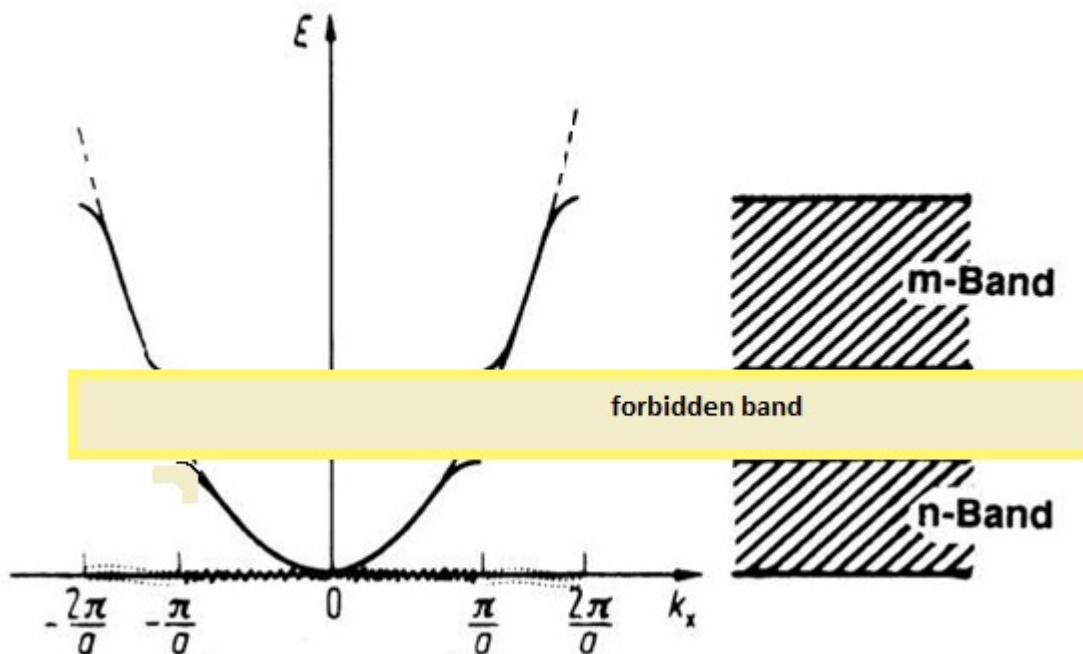
There are forbidden bands in the E axis where the electron cannot be present.

Explanation of the band structure: If one have N atoms with large distance among them, the atomic orbitals (e.g. 2s or 2p) have discrete energies. If one moves them closer and closer, decreasing the distance *a* among them, the wave functions overlap (touch each other). Because of the Pauli principle, the electrons of different atoms cannot be at the same quantum states (orbitals), therefore the wave functions have to change. It means that the energy levels are also changing and instead of one sharp level, now we have N closely packed levels, which can be considered as a band. So each level become a band, but when $a=a_0$ (the equilibrium lattice constant), these band may overlap.
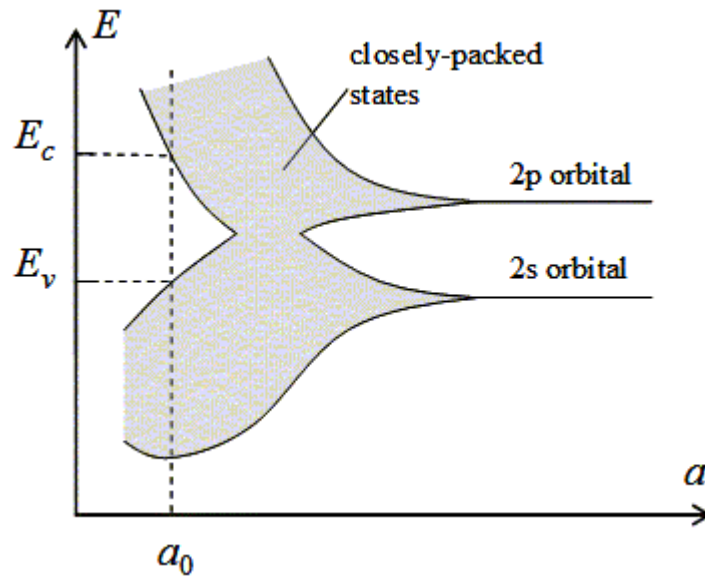
That is how the new quantum number n from the Bloch theorem can be derived from the quantum numbers of the single atom.

Video example:

https://www.youtube.com/watch?v=ots5zxbrlUk

Band structure and the classification of solids

In solid-state physics, the electronic band structure (or simply band structure) of a solid describes those ranges of energy that an electron within the solid may have (called energy bands, allowed bands, or simply bands) and ranges of energy that it may not have (called band gaps or forbidden bands). Band theory derives these bands and band gaps by examining the allowed quantum mechanical wave functions for an electron in a large, periodic lattice of atoms. Band theory has been successfully used to explain many physical properties of solids, such as electrical resistivity and optical absorption, and forms the foundation of the understanding of all solid-state devices (transistors, solar cells, etc.).

In solids, the **valence band** is the highest range of electron energies in which electrons are normally present at absolute zero temperature. It contains usually all electrons which participate in the chemical bonding between the atoms.

The **conduction band** is the lowest band which is not completely full.

Fully occupied band do not contribute to the conduction. The average velocity of their electrons is zero, regardless of the external field.
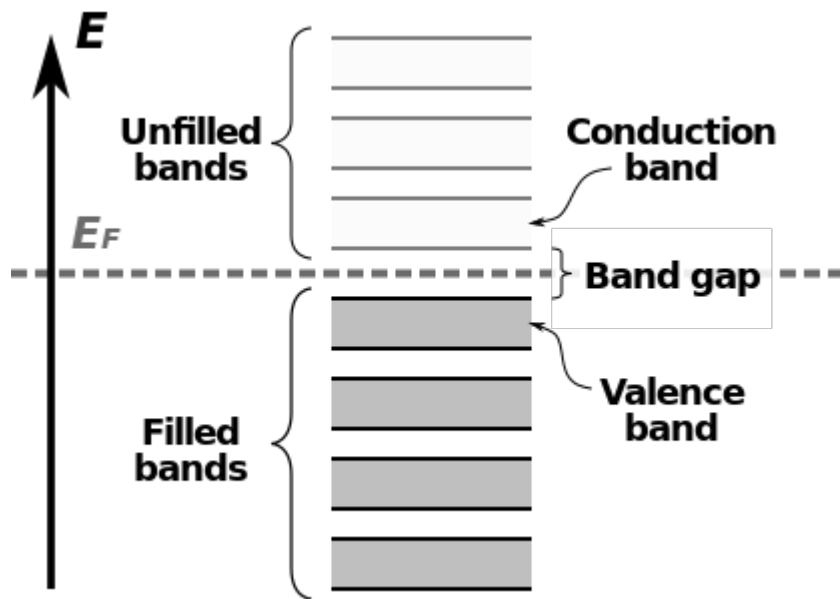
A useful way to visualize the difference between conductors, insulators and semiconductors is to plot the available energies for electrons in the materials. Crucial to the conduction process is whether or not there are electrons in the conduction band.

In <u>insulators</u> the electrons in the valence band are separated by a large gap from the conduction band. Photons of visible light cannot promote electrons into the conduction band, thus these insulators are transparent.

In <u>conductors</u> like metals, there is a band which is only partially filled, thus this is the valence band and the conduction band at the same time. The other possibility is that the valence band overlaps the conduction band. These materials are not transparent.

In <u>semiconductors</u> there is a small enough gap between the valence and conduction bands that thermal or other excitations can bridge the gap. With such a small gap, the presence of a small percentage of a doping material can increase the conductivity dramatically.
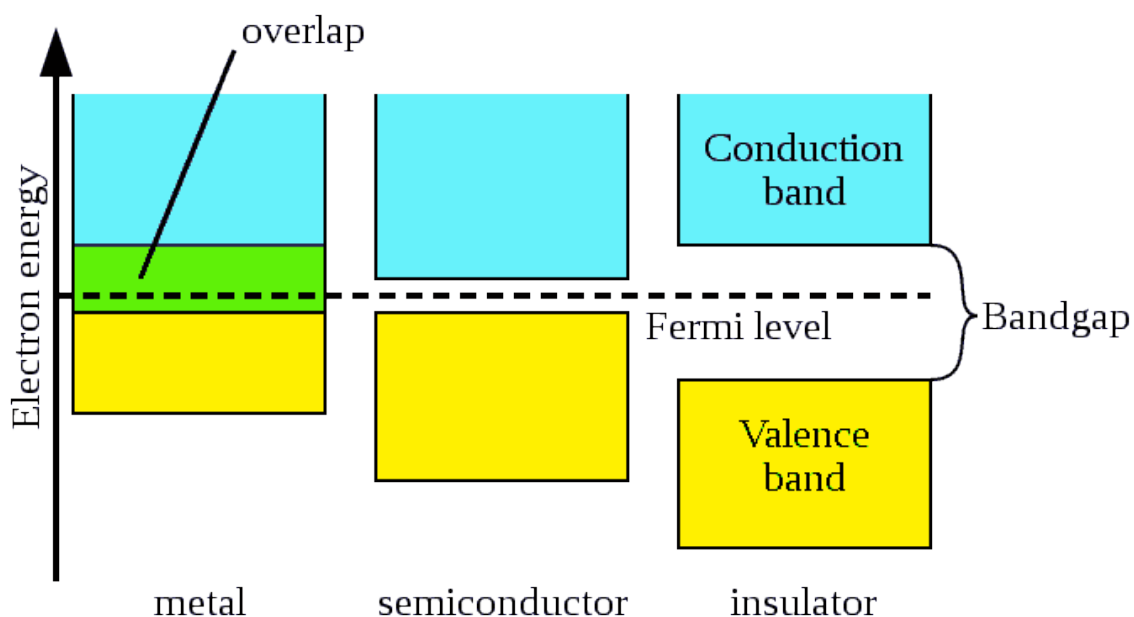

An important concept in the band theory is the Fermi level, the top of the available electron energy levels at low temperatures. The position of the Fermi level with the relation to the conduction band is a crucial factor in determining electrical properties.

Semiconductors

Intrinsic Semiconductors

An **intrinsic** semiconductor, also called an undoped semiconductor or i-type semiconductor, is a pure semiconductor without any significant dopant species or impurities. In an intrinsic semiconductor such as pure silicon at temperatures above absolute zero, there will be some electrons which are excited across the band gap into the conduction band and which can contribute to current flow. When the electron in pure silicon crosses the gap, it leaves behind an electron vacancy or "hole" in the regular silicon lattice. Under the influence of an external voltage, both the electron and the hole can move across the material. The number of charge carriers is determined by the properties of the material itself and the temperature, instead of the amount of impurities.
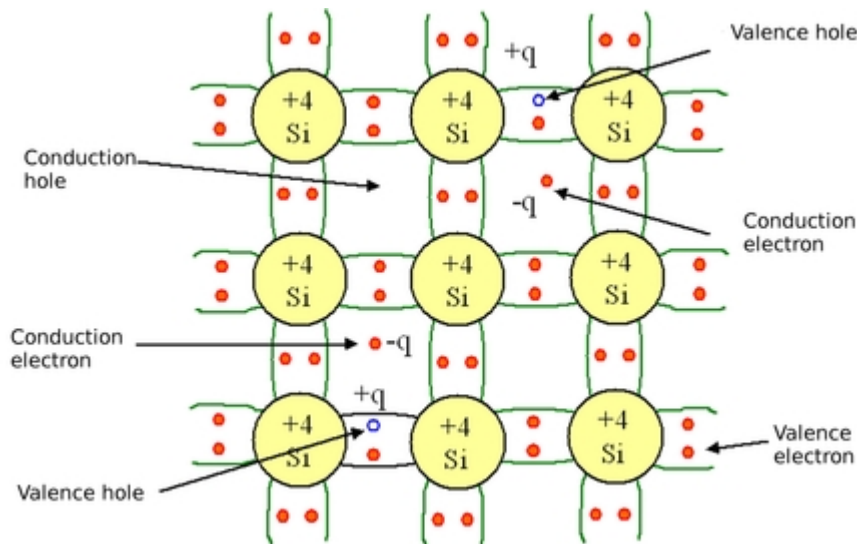
In intrinsic semiconductors the number of excited electrons and the number of holes are equal:

$$n_i = p_i \quad .$$

From the Fermi-Dirac distribution one can see that the number of electrons in the conduction band increases exponentially with the temperature. Because of the temperature new electron-hole pairs are continuously created. On the other hand, sometimes an electron meets a hole and fill it, which is called **recombination**. In thermal equilibrium the generation and recombination rates are equal, thus a dynamic equilibrium is reached.

Example: Suppose that the Fermi-level is 0.5eV lower than the bottom edge of the conduction band. At T=300K temperature,

$$kT = 1.38 \cdot 10^{-23} \frac{J}{K} \cdot 300K = 4.14 \cdot 10^{-21} J = 0.0259 \text{eV}$$

Using this we obtain

$$e^{\frac{E_i - E_F}{kT}} = e^{\frac{0.5}{0.0259}} = e^{19.3} = 2.46 \cdot 10^8$$

The +1 term is negligible in the denominator, thus the probability of occupation is:

$$f_{FD}(E) = \frac{1}{e^{\frac{E-E_F}{kT}}+1} \approx 4 \cdot 10^{-9}$$

If the temperature drop to 150K, half of the original value, the probability drops to

$$e^{-38,6} = 1.7 \cdot 10^{-17},$$

which means 8 orders of magnitude! This number is the same if we consider another semiconductor at 300K with twice larger gap.

Doped semiconductors

The application of band theory to n-type and p-type semiconductors shows that extra localized levels have been added by the impurities. The concentration of donors (or acceptors) are far too low to reach the point when the donor (or acceptor) states overlap, thus they do not contribute to the conduction directly.

In n-type material there are extra atoms added to the semiconductor with one more valence electron to create energy levels near the top of the band gap so that they can be easily excited into the conduction band. This shifts the effective Fermi level to a point about halfway between the donor levels and the conduction band.
In p-type material, extra holes in the band gap allow excitation of valence band electrons, leaving mobile holes in the valence band. This shifts the effective Fermi level to a point about halfway between the acceptor levels and the valence band.

n type   **p type**
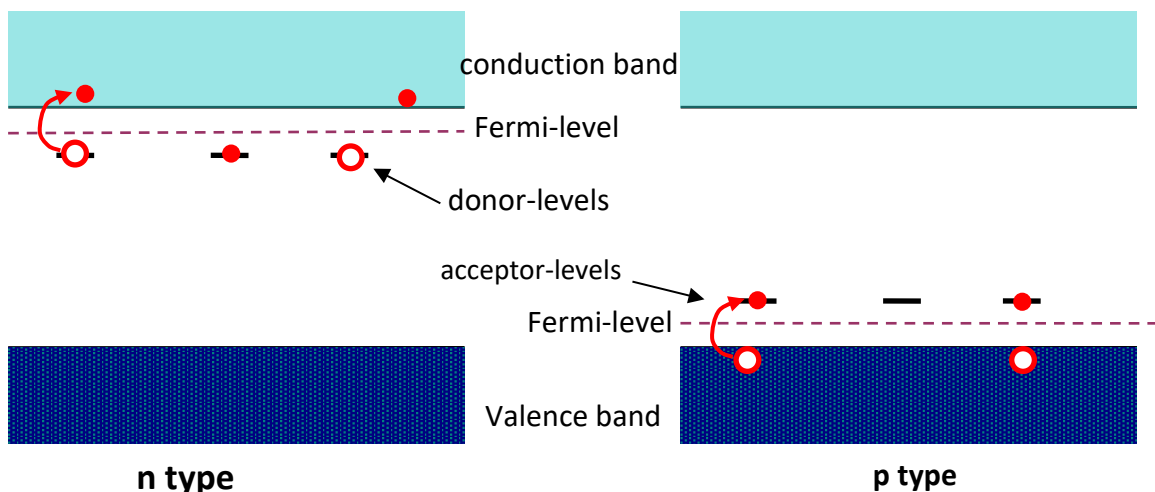
For the Group IV semiconductors such as silicon and germanium, the most common dopants are acceptors from Group III like boron or gallium and donors from Group V elements like arsenic, phosphorus.
In an n-type material, the concentration of the electrons (n) is larger than in an intrinsic material (n_i), thus the holes meet with electrons more frequently. This increase the probability of recombination, therefore decreasing p, the concentration of holes. The so called **law of mass action** is valid at thermal equilibrium:

$$n \cdot p = n_i \cdot p_i$$

The Hall-effect:

It is possible to distinguish between p and n type semiconductors by the **Hall effect**.

If an electric current flows through a conductor in a magnetic field, the magnetic field exerts a transverse force on the moving charge carriers which tends to push them to one side of the conductor. This is most evident in a thin flat conductor as illustrated. A build-up of charge at the sides of the conductors will balance this magnetic influence, producing a measurable voltage, the so called Hall-voltage between the two sides of the conductor. The presence of this Hall-voltage is called the Hall effect.
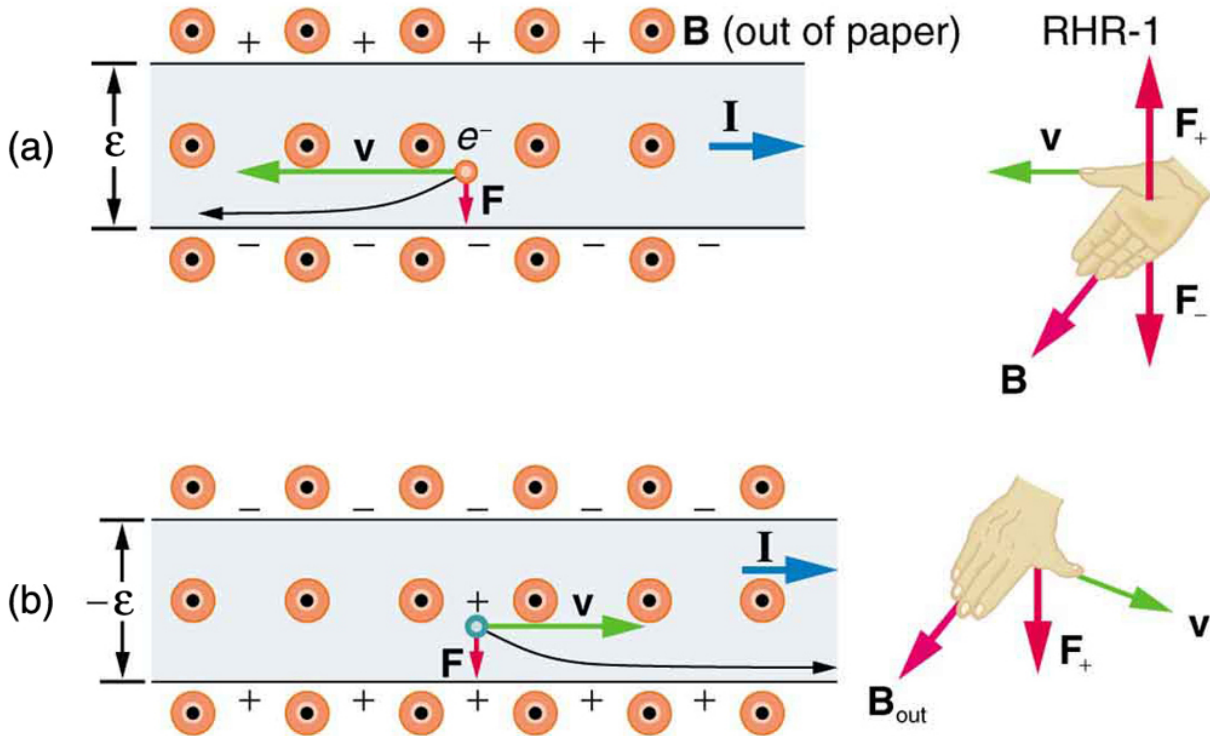

Hall-sensors: The Hall voltage is very sensitive to the magnetic field. If we move the magnet (the source of B) in the experiment above, the Hall voltage will change. One can use it to detect the position of the magnet and therefore to detect the position of any moving object contains a magnet. Using the position data one can detect velocity, frequency and rotational speed, for example in engines. Hall-effect sensors have many applications in computers (e. g. as reliable keyboard switches, i.e. detecting if you press a button in your keyboard) and cars (as fuel level indicators).

p-n junctions

A p–n junction is a boundary or interface between two types of semiconductor material, p-type and n-type, inside a single crystal of semiconductor.

After joining p-type and n-type semiconductors, electrons from the n region (majority carriers) near the p–n interface tend to diffuse into the p region. As electrons diffuse (because of concentration-difference), they leave positively charged ions (donors) in the n region. Likewise, holes from the p-type region (also majority carriers there) near the p–n interface begin to diffuse into the n-type region, leaving fixed ions (acceptors) with negative charge. The regions nearby the p–n interfaces lose their neutrality and become charged, thereby impeding the diffusion and   producing a drift current of minority carriers. In a p–n junction, without an external applied voltage, an equilibrium condition is reached in which a potential difference (the so called built-in potential V_{bi}) is formed across the junction to balance the diffusion current.
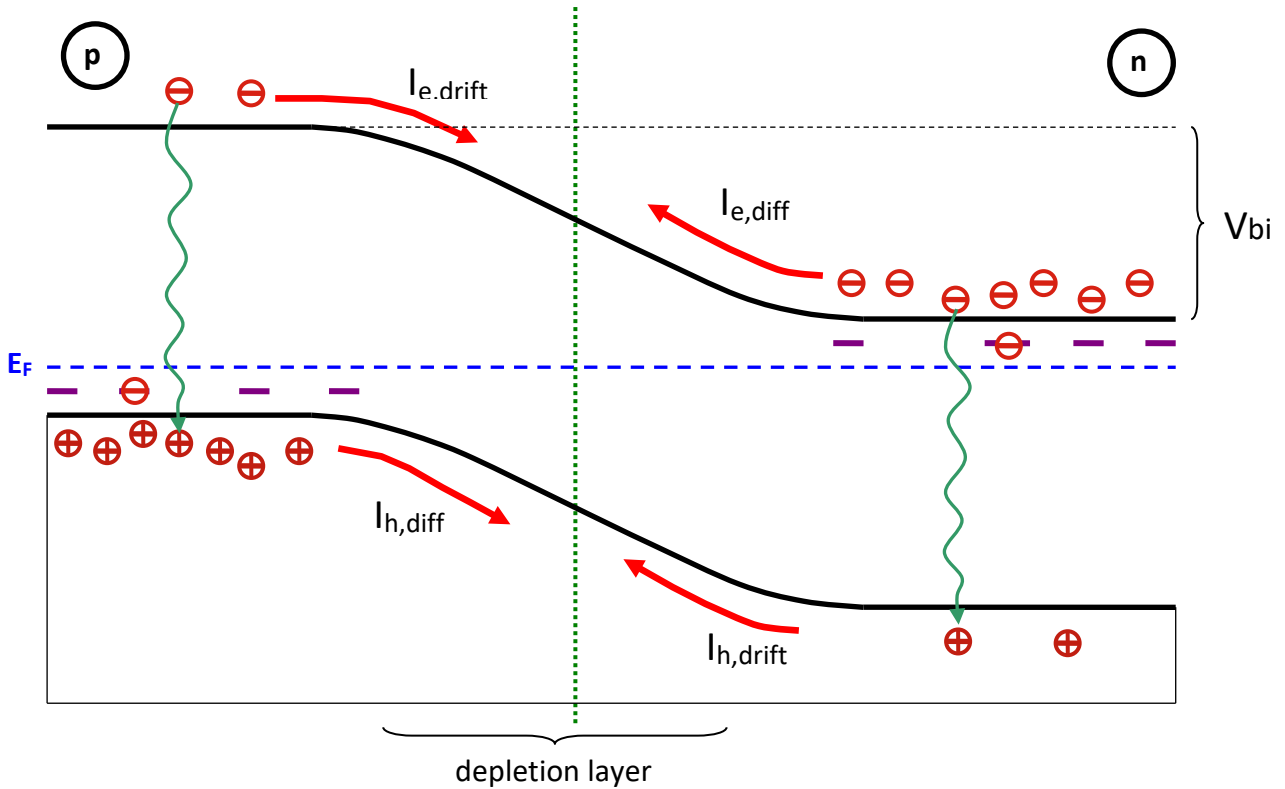
In dynamic equilibrium:

$$I_{e,drift} = I_{e,diff} \quad \text{and} \quad I_{h,drift} = I_{h,diff}$$

where for example $I_{e,drift}$ is the current of minority electrons caused by V_{bi} (sometimes denoted by V_0), while $I_{h,diff}$ is the current of majority holes produced by random diffusion motion of holes at the p side against the built-in potential.
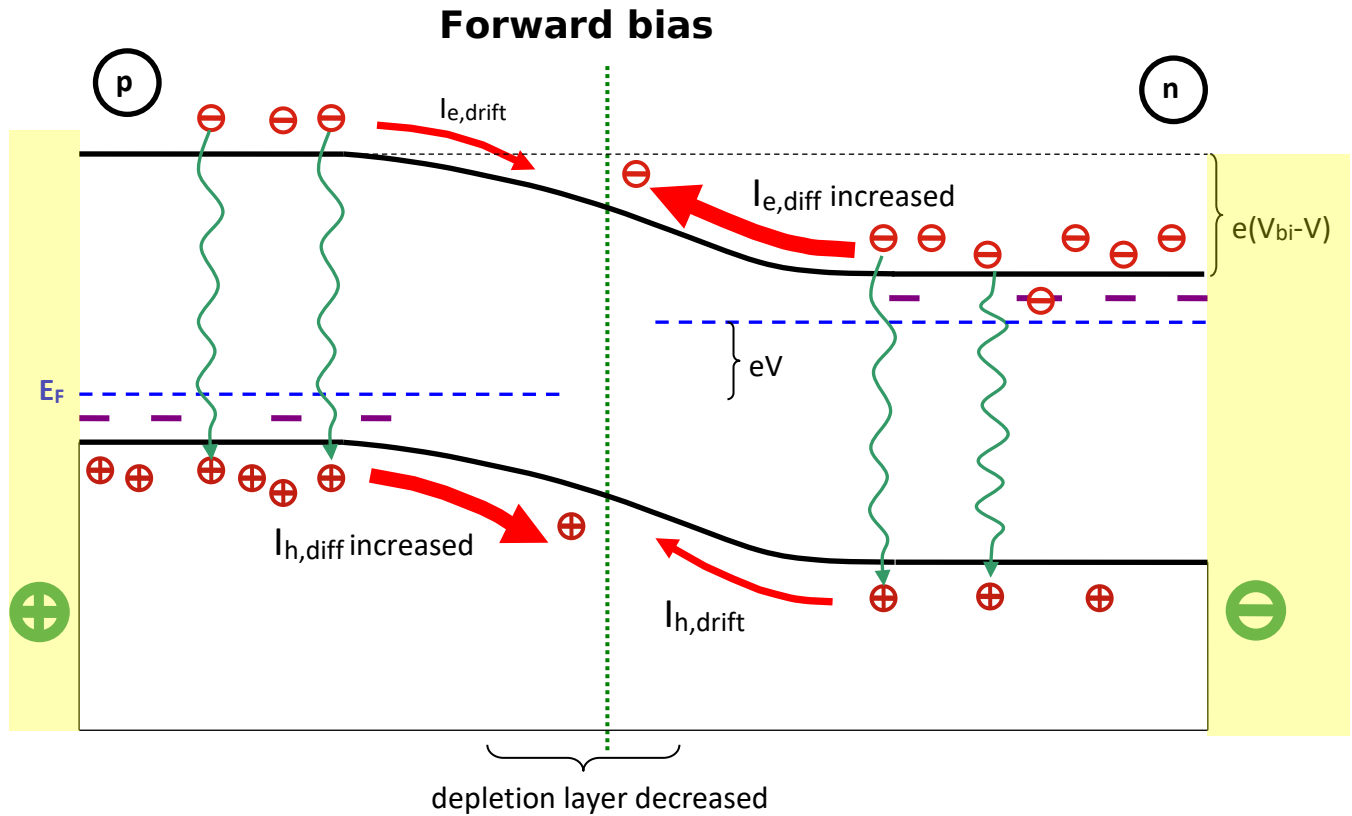
At the proximity of the junction, the electrons and holes meet and recombine with a very high probability, thus the concentration of charge carriers become low. We call this region space charge region or depletion layer.
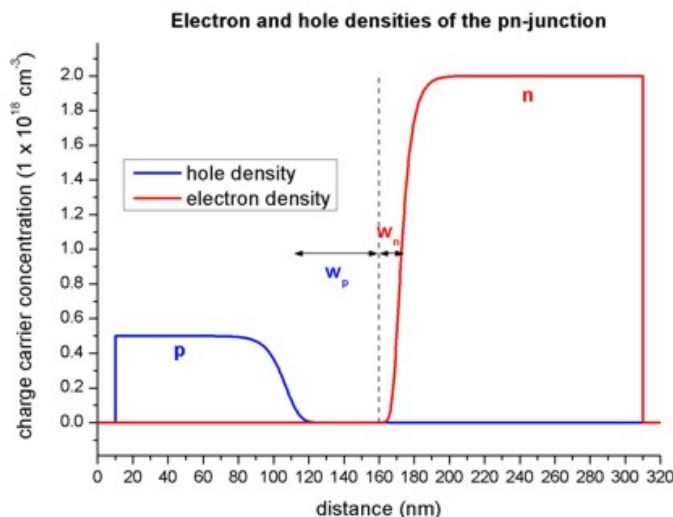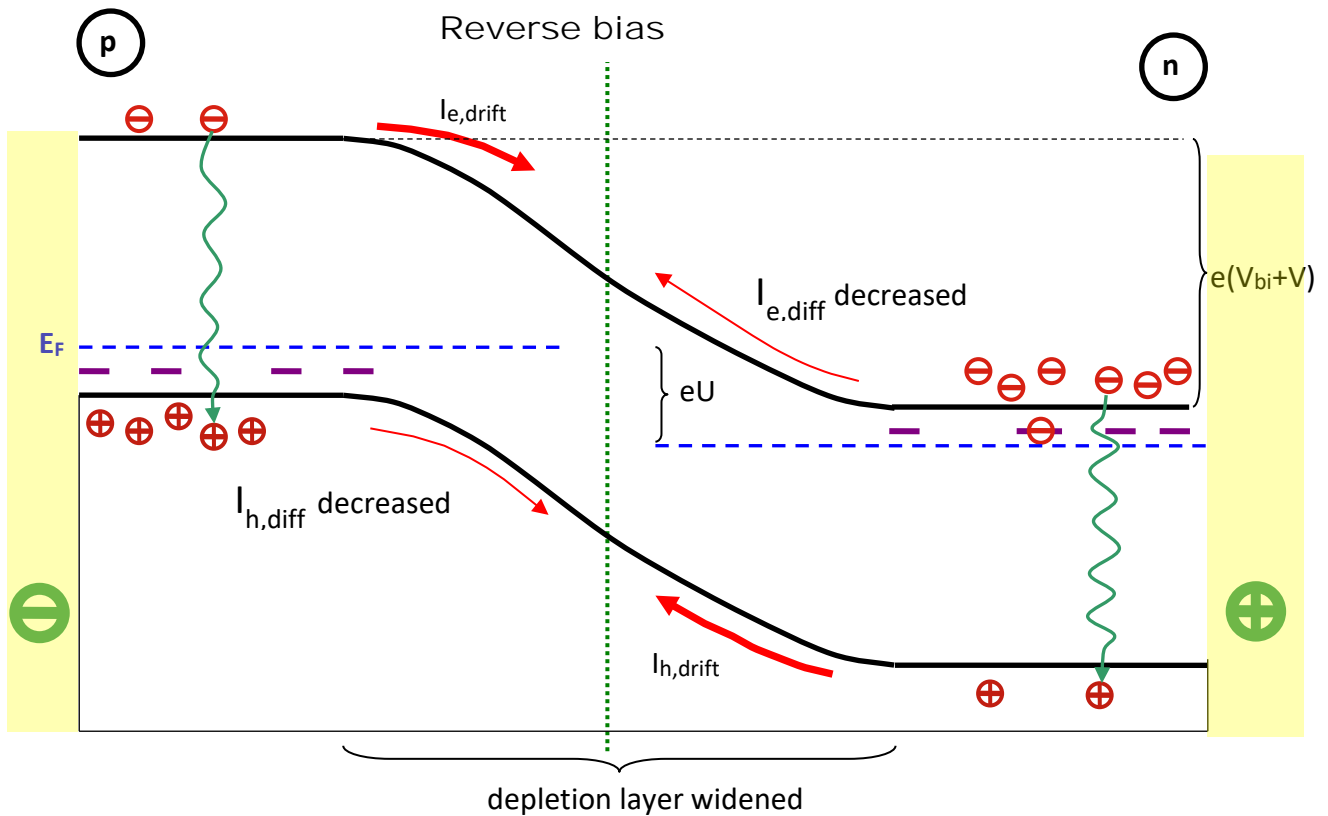
Diodes

If we apply <u>forward bias</u> to the junction, the negative pole of the battery is connected to the n side. This decreases the potential difference at the junction thus the diffusion motion of majority electrons and against the potential is increased. It means that a significant current is flowing.
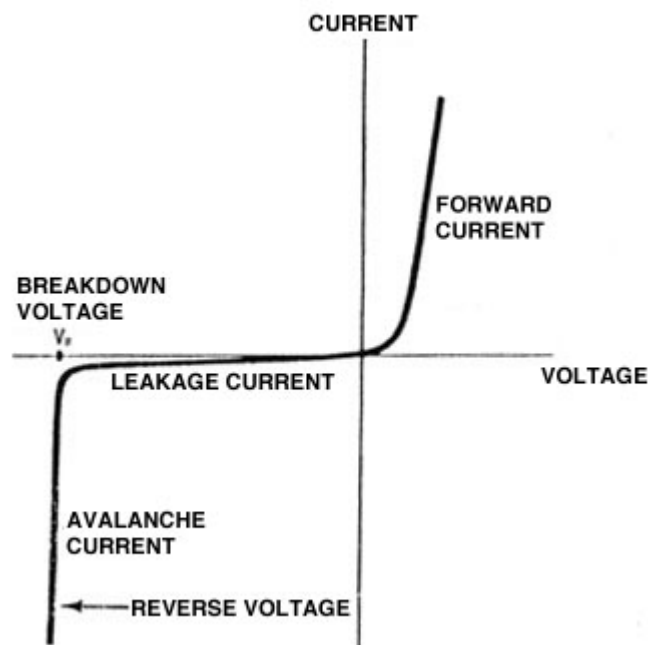
Forward bias

depletion layer decreased

The current does **not** obey Ohm's law, but Shockley's diode equation:

$$I \approx I_0 \cdot \left(e^{\frac{eV}{kT}} - 1 \right) \quad \text{(Shockley-law)}$$

In case of <u>reverse bias</u>, the positive terminal is connected to the n side of the junction. This attracts the electrons away from the junction and similarly, the holes are also repelled from the junction. It means that the depletion layer is widened. The potential barrier is increased, making the diffusion harder. On the other hand, the minority carriers can easily drift across the junction. However, the concentration of the minority carriers is low and nearly independent from the voltage, thus only a very weak current is flowing.

Reverse bias

p

n

$I_{e,drift}$

$I_{e.diff}$ decreased

E_F

eU

$I_{h,diff}$ decreased

$I_{h,drift}$

$e(V_{bi}+V)$

depletion layer widened

When the reverse bias becomes very large, reaching the so called breakdown voltage, the generation process of minority carriers accelerates leading to an avalanche condition which can cause runaway and even destroy the diode.


CURRENT

FORWARD CURRENT

BREAKDOWN VOLTAGE
V_z

LEAKAGE CURRENT

VOLTAGE

AVALANCHE CURRENT

←—REVERSE VOLTAGE

PN junctions and the light

1. **Light Emitting Diodes** (LEDs) are semiconductor devices which turn electric energy into optical energy. LEDs are essentially forward biased p-n junctions and when electrons cross the junction from the n- to the p-type material, the electron-hole recombination process can

produce some photons in the IR or visible range of the spectrum in a process called electroluminescence, thus and in this case the semiconductor surface emits light.

As the momentum of the photon is very small, the phonon-generation has significant probability only if the electrons at the lowest part of the conduction band has the same momentum as the top of the valance band. If this is the case, we talk about **direct band gap** semiconductors. In case of indirect band gap, an additional particle (e.g. phonon[1]) must be created at the same time with the photon, which has very low probability, thus usually only phonons are created.

transition in case of
direct band-gap (e.g. GaAs)

transition in case of
indirect band-gap (e.g. Si, Ge)
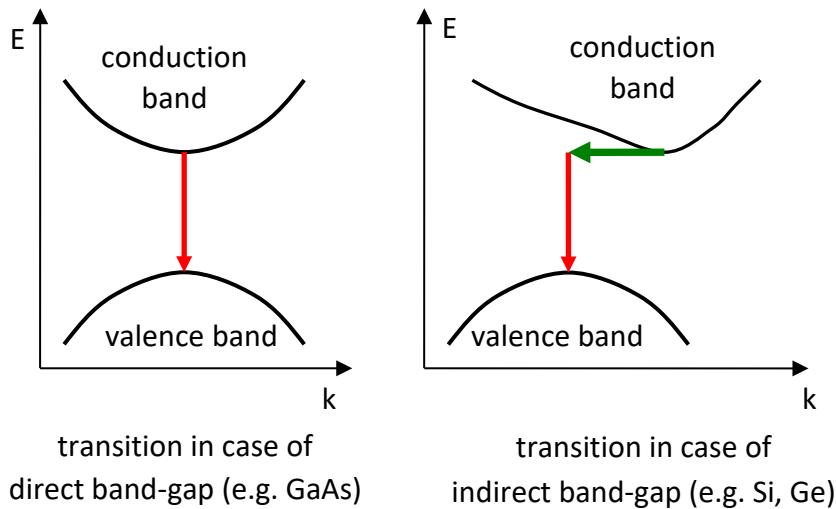
LEDs are constructed of gallium arsenide (GaAs), gallium arsenide phosphide (GaAsP), or gallium phosphide (GaP) because they have direct band gaps. Silicon and germanium are not suitable because they have indirect band gap and thus those junctions produce heat and no appreciable infrared or visible light.

2. A solar cell, or photovoltaic cell, is an electrical device that converts the energy of light directly into electricity by the photovoltaic effect. It is an unbiased p-n junction, where the absorbed photon energy generates electron-hole pairs. Because of the built-in potential, the generated electrons flow towards the n edge of the junction and holes flow to the p edge, and the junction becomes a source of electricity. So the built-in potential is the major driving force of the spatial motion, but it is the energy of the photons which excite the electrons into the high potential states.

Photons incident on the surface of a semiconductor will be either reflected from the top surface, will be absorbed in the material or, failing either of the above two processes, will be transmitted through the material. For photovoltaic devices, reflection and transmission are typically considered loss mechanisms as photons which are not absorbed do not generate power. If the photon is absorbed it has the possibility of exciting an electron from the valence band to the conduction band. Photons falling onto a semiconductor material can be divided into three groups based on their energy compared to that of the semiconductor band gap:

   a) Photons with energy less than the band gap energy interact only weakly with the semiconductor, passing through it as if it were transparent.

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[1] In quantum mechanics, the oscillations of atoms are also quantized. A **phonon** is an elementary quantum of the vibrational motion in which a lattice of atoms or molecules uniformly oscillates at a single frequency.

b) Photons with just slightly more energy than the band gap have enough energy to create an electron hole pair and are efficiently absorbed.

c) Photons with energy much greater than the band gap are strongly absorbed, but the electron is excited not to the bottom of the conduction band, but to states with much higher energy. Thus for photovoltaic applications, a significant part of the energy of these photons is wasted as electrons quickly thermalize back down to the bottom of the conduction band.

If the band-gap is wide, loss mechanism a) is large, but decreasing the band gap means increasing losses explained at c). Because of these losses, traditional single-junction cells have a maximum theoretical efficiency of 34%. That is why **multi-junction** (MJ) solar cells are developed, with multiple p–n junctions made of different semiconductor materials. Each material's p-n junction will produce electric current in response to different wavelengths of light. The use of multiple semiconducting materials allows the absorbance of a broader range of wavelengths, improving the cell's sunlight to electrical energy conversion efficiency.


(a) (b)
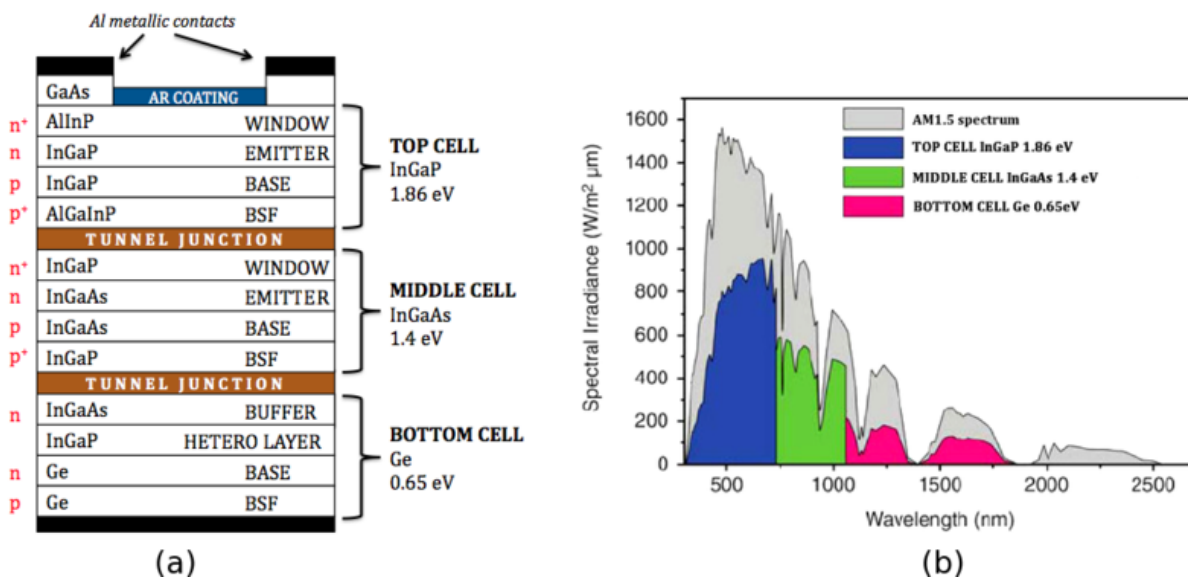
When the light reach the top of the multi-junction solar cell, only high energy photons are absorbed with small c) type loss, the others reach the next junction with smaller band gap, etc. Lab examples of multi-junction cells have already demonstrated performance over 43%.
However, the generated current have to go across all the junctions. Between the cells, special quantum-tunnel junctions are used to reduce the resistance of the device.

Tunnel diode

The tunnel diode has a region in its voltage current characteristic where the current decreases with increased forward voltage, known as its negative resistance region. This characteristic makes the tunnel diode useful in oscillators and as a microwave amplifier. Quantum mechanical tunneling is responsible for the phenomenon and thus this device is named as tunnel diode.
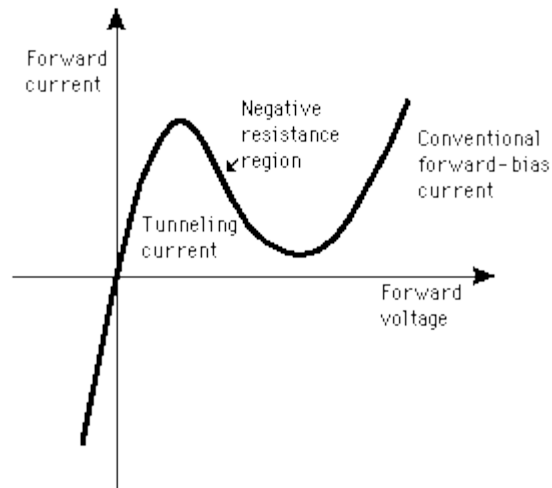
The concentration of dopants in both p and n region is very high and the p-n junction is abrupt. For this reasons, the depletion layer width is very small. Because of high doping, the bottom of the conduction band is full of electrons in the n side while the top of the valance bend is full of holes in the p side. The Fermi energy level lies in the valence band on the p side of the junction and the conduction band on the n side. Therefore, electrons can tunnel from the valence band to the conduction band without changing energy. When there is no potential voltage across the diode (fig. a), zero net current flows since tunnelling may initially occur in either direction with the same probability.
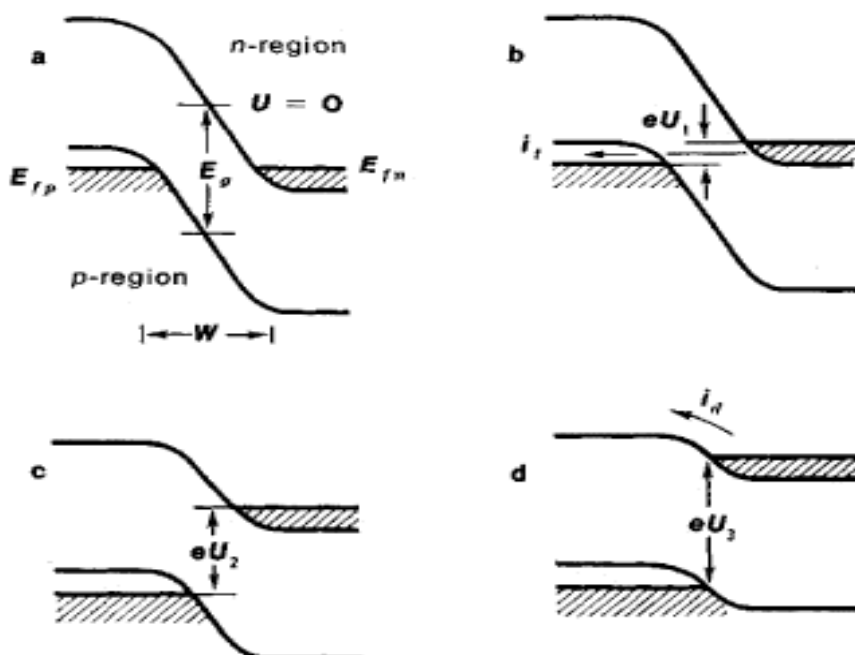

As the voltage increases (fig. b), the electrons can tunnel more easily to the p side (a similar phenomenon occurs also for the reverse bias, that is why the large reverse current). This initially causes increased tunnelling and a resulting current. At a specific voltage the bottom of the n-side conduction bands get higher than the top of the p-side valance band, which makes tunnelling impossible. However, as the voltage increases, the usual (conventional) diffusion current increases exponentially (fig. d) and the diode behaves like ordinary diodes.

Transistors

A transistor is a semiconductor device used to amplify and switch electronic signals and electrical

power. It is composed of semiconductor material with at least three terminals for connection to an external circuit. A voltage or current applied to one pair of the transistor's terminals changes the current through another pair of terminals. Because the controlled (output) power can be higher than the controlling (input) power, a transistor can amplify a signal. Today, some transistors are packaged individually, but many more are found embedded in integrated circuits. The transistor is the fundamental building block of modern electronic devices, and is ubiquitous in modern electronic systems.

Bipolar junction transistor:
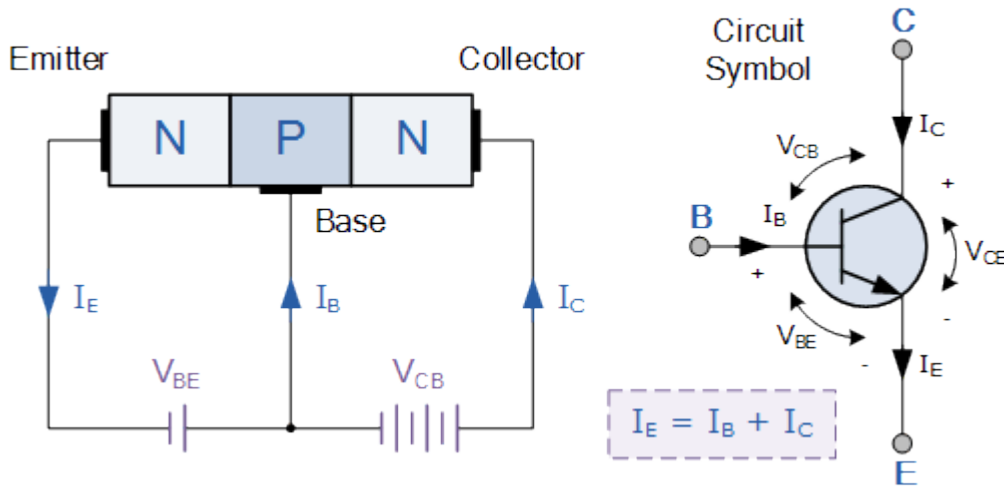
It is customary to reverse bias the base-collector junction of a bipolar junction transistor. Note that this increases the width of the depletion region at the junction between the base and the collector. The reverse bias voltage could be a few volts to tens of volts for most transistors. There is no current flow, except leakage (minority) current, in the collector circuit.

Let us examine when a voltage source has been added to the emitter base circuit. Normally we forward bias the emitter-base junction, overcoming the V_{bi} potential barrier. This is similar to forward biasing a junction diode. The majority carriers (electrons for n-p-n) are flowing from the emitter into the base and becomes minority carriers there.

If the base region were thick and/or heavily doped, as in a pair of back-to-back diodes, all the current entering the base would flow out the base lead. In our n-p-n transistor example, electrons leaving the emitter for the base would recombine with holes in the base, making room for more holes to be created at the (+) battery terminal on the base as electrons exit.

However, the base is manufactured thin and lightly doped. Only a few majority carriers in the emitter, injected as minority carriers into the base, actually recombine and flow directly through the base to the positive battery terminal. Most of the emitter current of electrons diffuses through the thin base into the collector. Moreover, modulating the small base current produces a larger change in collector current. If the base voltage falls below approximately 0.6 V for a silicon transistor, the large emitter-collector current drastically decreases.

MOSFETs

The field-effect transistor (FET) is a transistor that uses not electric current but electric field to control the shape and hence the conductivity of a channel in a semiconductor material. One possibility to achieve this is to place an **insulating layer** between the gate and the channel. This allows for a wider range of control (gate) voltages and heavily decreases the gate current and thus the input power (the input resistance of a MOSFET is usually greater than 10^{12} Ohms) compared to the bipolar junction transistor. The insulator is typically made of an oxide (such as silicon dioxide, SiO_2), that is why we use the phrase MOSFET (Metal-Oxide-Semiconductor Field Effect Transistor).

There are two classes of MOSFETs. There are depletion mode and there are enhancement mode MOSFETs. Each class is available as n-channel (where the majority carriers are electrons) or a p-channel (where the majority carriers are holes), giving a total of four types of MOSFETs. An **enhancement mode** device (also called a ***normally off*** MOSFET) is built without a channel and does not conduct current when V_{GS} = 0.
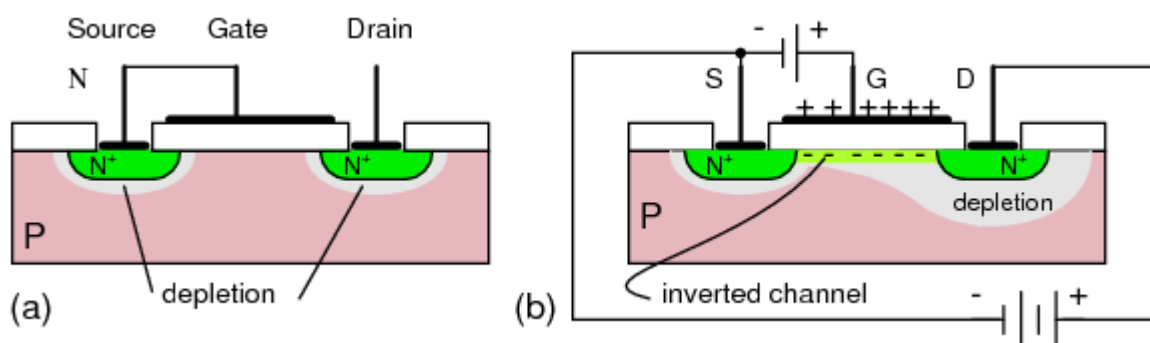
If we apply a positive voltage to the gate we'll set up an electrostatic field between it and the rest of the transistor. The positive gate voltage will push away the (majority) holes inside the p-type substrate and attracts the moveable (minority) electrons from the substrate. This produces a layer just under the gate's insulator in which the type of doping is effectively inverted and through which electrons can get into and move along from source to drain without significant recombination. The positive gate voltage therefore 'creates' a channel in the top layer of material. Increasing the value of the positive gate voltage pushes the p-type holes further away and enlarges the thickness of the created channel. As a result we find that the size of the channel we've made increases with the size of the gate voltage and enhances or increases the amount of current which can go from source to drain — this is why this kind of transistor is called an enhancement mode device.

A **depletion mode** device has a built-in channel in resting state, thus this device conducts current when no bias applied. The channel gets smaller as a reverse bias (negative in case of n-channel) is applied to the Gate and decrease the I_{SD} current.

Both n and p channel MOSFETs can be connected in parallel and in series to implement logical gates.
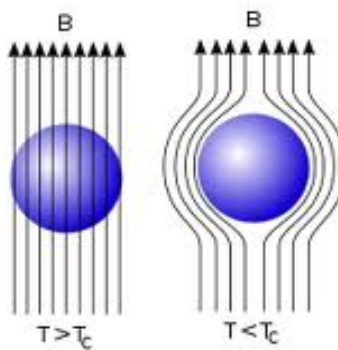
Superconductivity

If mercury is cooled below 4.1 K, it loses all electric resistance for direct current. This discovery of

superconductivity by H. Kammerlingh Onnes in 1911 was followed by the observation of other metals which exhibit **zero direct current resistivity** below a certain critical temperature. It also means that there is no power loss in the material. The fact that the resistance is zero has been demonstrated by sustaining currents in superconducting lead rings for many years with no measurable reduction. An induced current in an ordinary metal ring would decay rapidly from the dissipation of ordinary resistance (power loss $P=I^2R$), but superconducting rings had exhibited a decay constant of over a billion years!

However, not all zero-resistance case can be called superconductors. In case of superconductors there is a well-defined phase transition at the critical temperature T_C and other properties of matter must be present.

One of the important properties of a superconductor is that it will exclude magnetic induction, a phenomenon called the **Meissner effect**.

$$\vec{B} = 0 \ \Rightarrow \ \vec{M} = -\vec{H} \ \text{ which means } \ \chi = -1$$

If we would like to increase the magnetic induction inside the superconductor, such macroscopic persistent current will be induced that the induction is completely cancelled out (Lenz law). Sometimes superconductors are called perfect diamagnets. This has an interesting consequence: magnetic levitation.
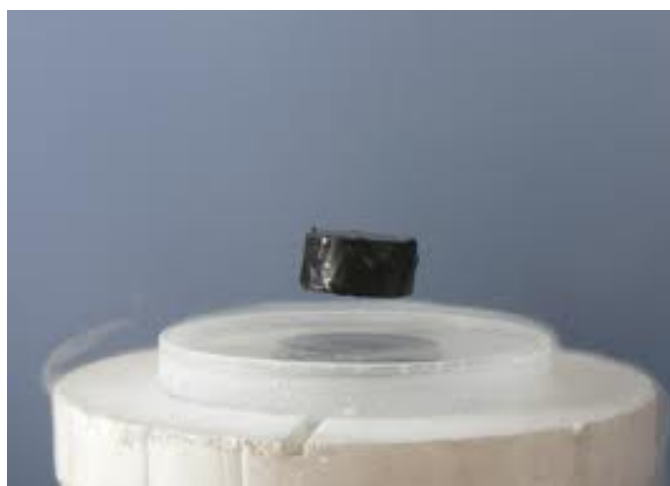

The superconducting state cannot exist in the presence of a magnetic field greater than a critical value, even at absolute zero temperature. This critical magnetic field is strongly correlated with the critical temperature for the superconductor.

$$H_C(T) = H_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right],$$

where $H_c(0)$ is the critical field at T=0K. The supercurrents have also their own magnetic field, thus for a given superconducting wire there is an upper limit for the current as well.

The situation is so simple only for the so called type I superconductors.
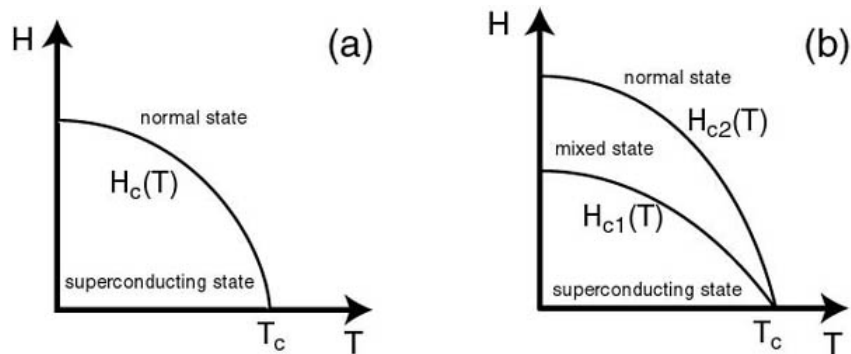

Type II superconductors show two critical magnetic field values, one at the onset of a mixed superconducting and normal state and one where superconductivity ceases. Above H_{C1} type II superconductors exist in a **mixed** or **vortex state** with normal cores surrounded by superconducting regions.
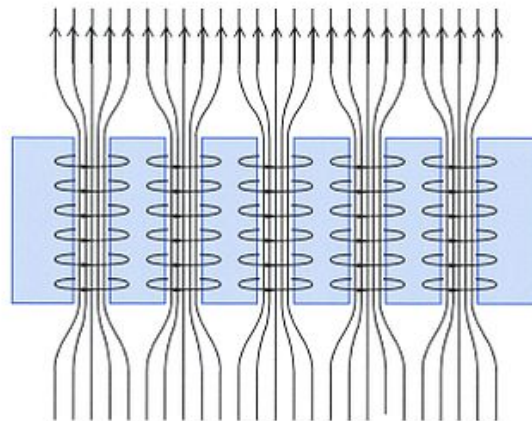
At H_{C1} in a Type II superconductor, magnetic fields begin to penetrate through cores of normal (i. e. not superconducting) material surrounded by superconducting current vortices. As long as these vortices are stationary (pinned), the magnetic fields can penetrate while still maintaining zero electric resistivity paths through the material. A size of about 300 nm is typical for the normal cores.

While the Meissner effect is modified to allow magnetic fields through the normal cores, magnetic fields are still excluded from the superconducting regions. H_{C2} can be quite large for some type II material, which makes the application of these superconductors much easier.

As the external magnetic field is increased, the vortices are packed closer together. The vortices feel a force when current flows, and if they move, heat is absorbed and the superconducting state can be lost. Microscopic defects can act to pin the vortices and maintain the superconducting state to a higher temperature. So the microscopic structure and fabrication techniques influence their properties greatly.
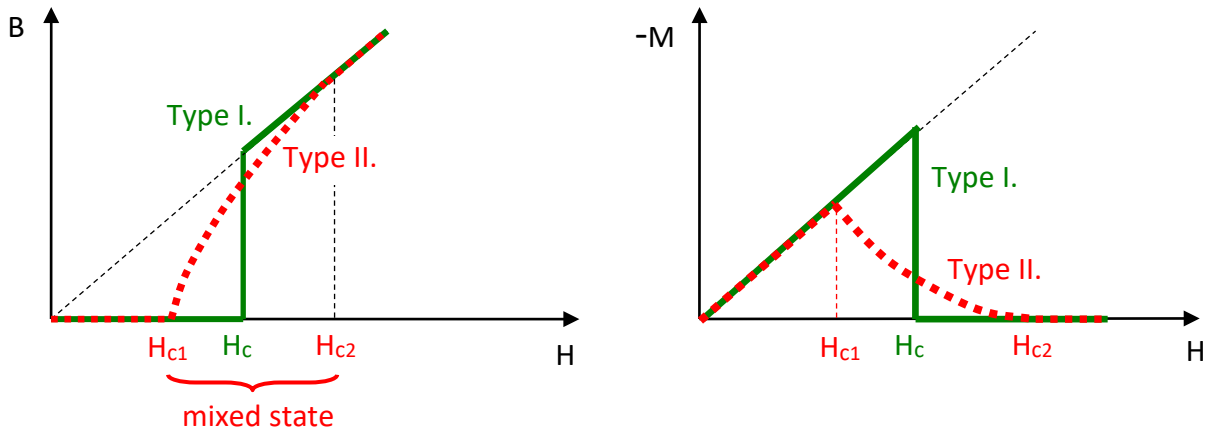
The figure shows the magnetic induction and the negative of the magnetization as a function of the field. The green lines are for type I, while the red dotted lines are for type II superconductors.

Theory of superconductivity:

In superconductors, opposite-spin electron **pairs** (called Cooper pairs) are coupling over a range of hundreds of nanometres, three orders of magnitude larger distance than the lattice spacing. These electron-couples have integer spins, thus they obey the Bose-Einstein statistics (no Pauli-principle), they take the character of a **boson** and all pairs condense into the lowest energy ground state.

As we explained above, electrical resistance in metals arises because electrons propagating through the solid are scattered due to deviations from perfect translational symmetry. These are produced e.g. by impurities (giving rise to a temperature independent contribution to the resistance) or the phonons - lattice vibrations - in a solid.

In a superconductor below its transition temperature Tc, there is no resistance because these scattering mechanisms are unable to impede the motion of the Cooper pairs. The mechanism by which two negatively charged electrons are bound together is still controversial in "modern" superconducting systems such as the copper oxides, but well understood in conventional superconductors such as aluminium in terms of the BCS (Bardeen Cooper and Schrieffer) theory, which is mathematically far too difficult to study here. The attraction is mediated by phonons, the quanta of lattice vibrations. To be as simple as possible, we can say that a passing electron attracts the lattice, causing a slight ripple toward its path which attracts the other electron.

The essential point is that below Tc the binding energy of a pair of electrons causes the opening of a **gap** in the energy spectrum at the Fermi energy, separating the pair states from the "normal" single electron states. The space occupied by one pair contains many other pairs, and there is thus a complex interdependence of the occupancy of the pair states. There is then insufficient thermal energy to scatter the pairs, as reversing the direction of travel of one electron in the pair requires the destruction of the pair and many other pairs due to the nature of the many-electron BCS wavefunction. The pairs thus carry current unimpeded.

A Josephson junction is a thin, usually insulating layer (so called weak link) between two superconductors. Cooper-pairs can go across this junction even without external electric field by quantum-tunneling. We call this phenomenon Josephson-effect.

C

A   B
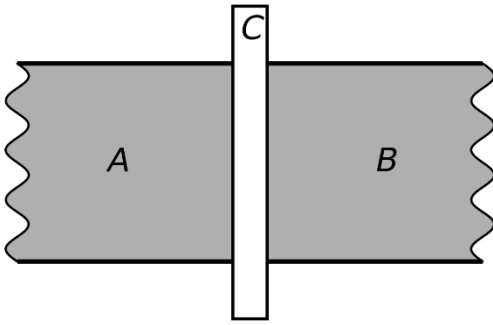
Applications of superconductors:

An idealized application for superconductors is to employ them in the transmission of electric power to cities. However, due to the high cost and impracticality of cooling miles of superconducting wire to low temperatures, this is still not widespread. Since 2001 some residents of Denmark and the USA started to receive electricity through high T_C superconducting wires. These wires carry much more current than a copper cable with the same cross section or weight.

Magnetic-levitation is an application where superconductors perform extremely well. Transport vehicles such as trains can be made to "float" on strong superconducting magnets, virtually eliminating friction between the train and its tracks. Not only would conventional electromagnets waste much of the electrical energy as heat, they would have to be physically much larger than superconducting magnets. A landmark for the commercial use of MAGLEV technology occurred in 1990 when it gained the status of a nationally-funded project in Japan. The Minister of Transport authorized construction of the Yamanashi Maglev Test Line which opened on April 3, 1997. In April 2015, the MLX01 test vehicle (shown above) attained an incredible speed of 374 mph (603 km/h).

Magnetic Resonance Imaging (MRI): An area where superconductors can perform a life-saving function is in the field of biomagnetism. Doctors need a non-invasive means of determining what's going on inside the human body. If a strong superconductor-derived magnetic field penetrates into the body, hydrogen atoms that exist in the body's water and fat molecules are forced to accept energy from the magnetic field. They then release this energy at a frequency that can be detected and displayed graphically by a computer.

Fault current limiter: Recently, power utilities have also begun to use superconductor-based "fault limiters". If the current is too high, it destroys superconductivity and the device appears in the circuit as a large resistor, thereby decreasing the current. This new generation of High T_C superconducting fault limiters is being called upon due to their ability to respond in just thousandths of a second to limit tens of thousands of amperes of current.

Computers: A petaflop is a thousand-trillion (10^{15}) floating point operations per second. Today's

fastest computers have only recently reached these speeds. Currently the fastest is operating at 16.32 petaflops per second. The fastest single processor is running at 8 teraflops. Some scientists think that devices on the order of 50 nanometres in size along with unconventional switching mechanisms, such as the Josephson junctions associated with superconductors, will be necessary to achieve the next level of processing speeds. Researchers have quantified this further by predicting that 100 billion superconducting Josephson junctions on 4000 microprocessors will be necessary to reach 32 petaflops per second. These Josephson junctions are incorporated into field-effect transistors which then become part of the logic circuits within the processors. Recently it was demonstrated that the tiny magnetic fields that penetrate Type 2 superconductors can be used for storing and retrieving digital information. It is, however, not a

foregone conclusion that computers of the future will be built around superconducting devices, because competing technologies also have the potential to achieve petaflop benchmarks.

Graphene and Silicene

Graphene (/ˈgræf.iːn/) is an allotrope of carbon in the form of a two-dimensional, atomic-scale, hexagonal lattice in which one atom forms each vertex. It is the basic structural element of other allotropes, including graphite, carbon nanotubes and fullerenes. It can also be considered as a (possibly indefinitely) large aromatic molecule, the limiting case of the family of flat polycyclic aromatic hydrocarbons.

Each carbon atom has a total of 6 electrons; 2 in the inner shell and 4 in the outer shell. The 4 outer shell electrons in an individual carbon atom are available for chemical bonding, but in graphene, each atom is connected to 3 other carbon atoms on the two dimensional plane, leaving 1 electron freely available in the third dimension for electronic conduction.
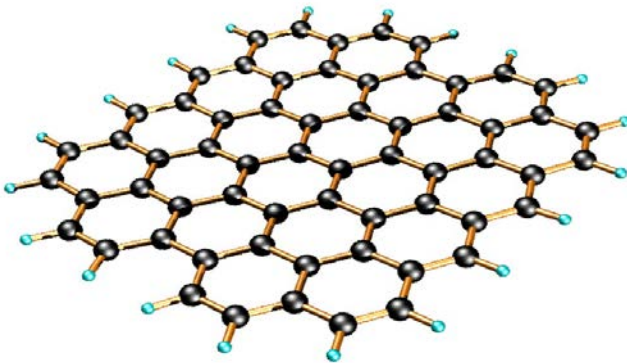
Graphene has many extraordinary properties. It is about 100 times stronger than steel by weight, conducts heat and electricity with great efficiency and is nearly transparent.
One of the most useful properties of graphene is that it is a zero-overlap semimetal (with both holes and electrons as charge carriers) with very high electrical conductivity. Unlike the band structure of semiconductors, for example, graphene has no band gap – no gap in energy between the electron-filled valence band and the unoccupied conduction band. In graphene these bands are represented by two cones ("Dirac cones") whose points touch, crossing linearly at the Dirac point. The Dirac point is a unique feature of graphene's band structure. When the valence band of graphene is completely filled and the conduction band is completely empty, the graphene can be considered "undoped". The free electrons close to the Dirac-point are located above and below the graphene sheet and are extremely mobile.
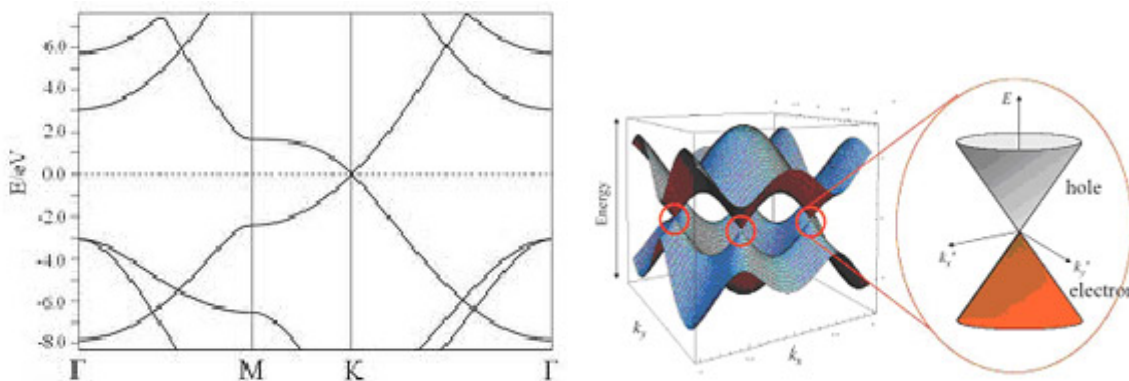
Native graphene has a zero energy gap and it is therefore not suitable as a channel material for FETs in digital electronics. In time, several options have been considered for opening a band gap in graphene, including cutting graphene flakes in narrow nanoribbons.
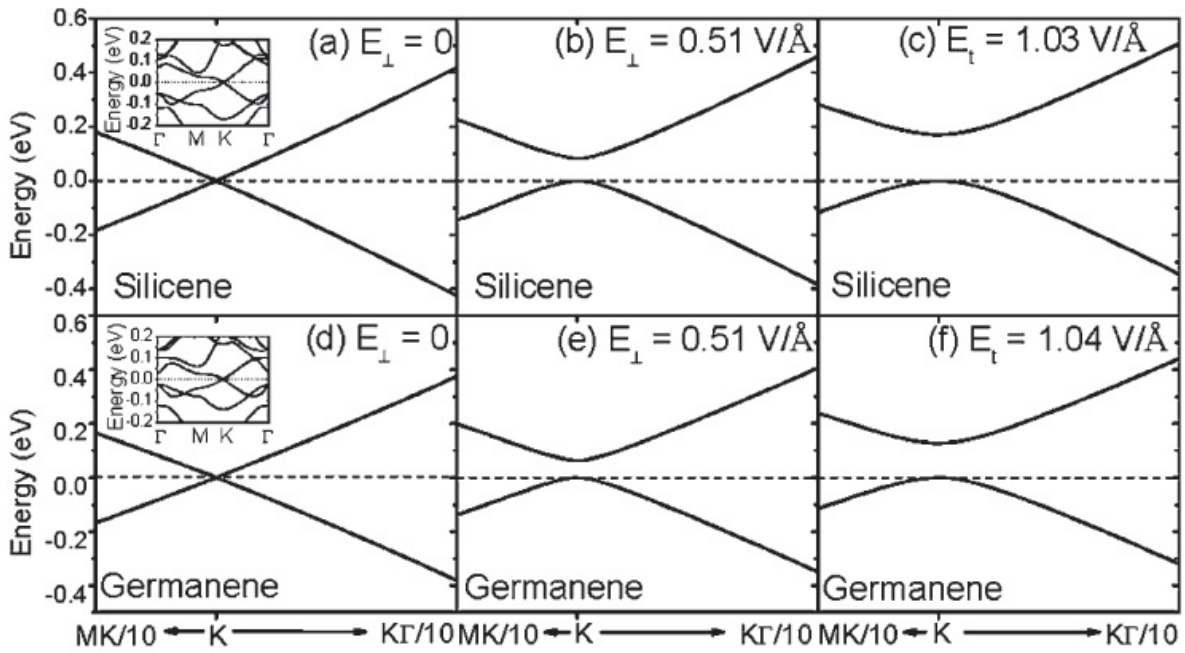
Silicene:

Although theorists had speculated about the existence and possible properties of silicone (and germanene), researchers first observed silicon structures that were suggestive of silicene only in 2010.

Silicon and carbon are similar atoms. They lie next to each other in the same group on the periodic table and have an $s^2 p^2$ electronic structure. Unfortunately silicene is not as stable as graphene, for example it rapidly degrades in air. The 2D structures of silicene and graphene are also quite similar, but have important differences. While both form hexagonal structures, graphene is completely flat, while silicene forms a buckled hexagonal shape.
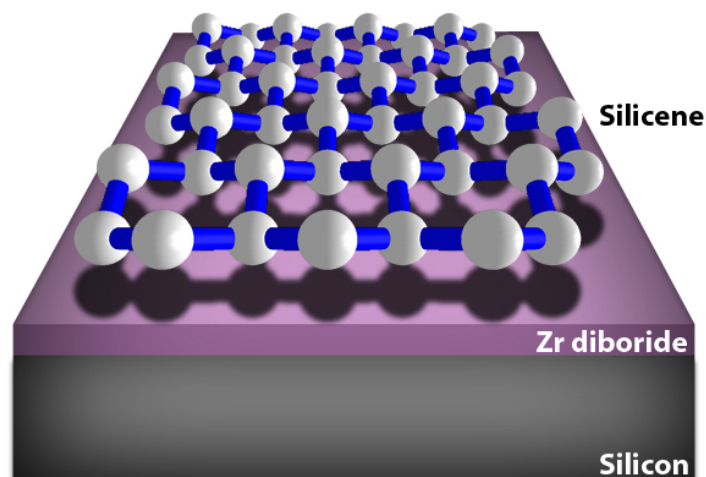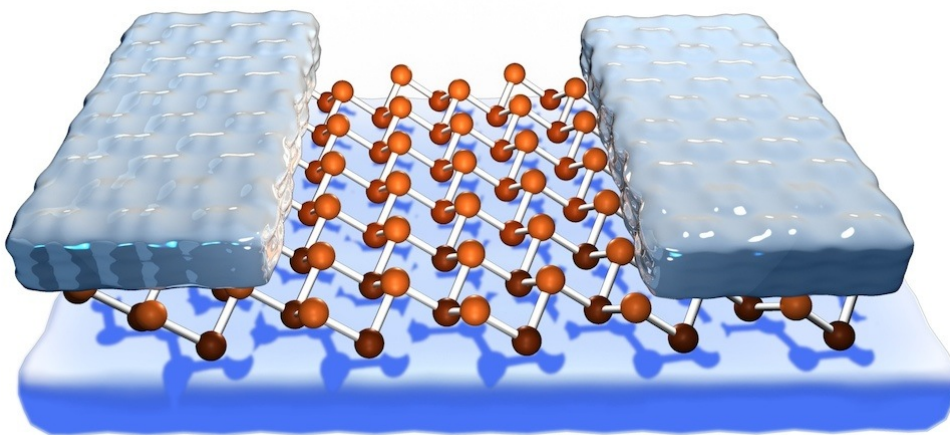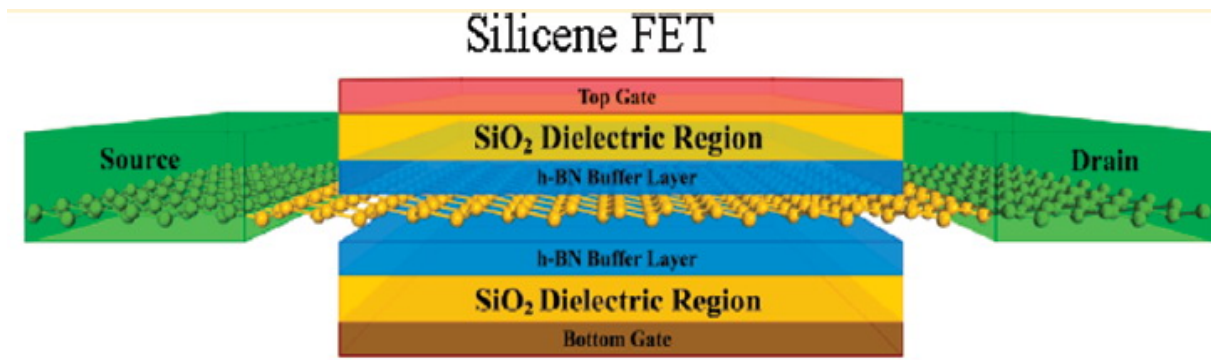

Silicene and graphene have similar electronic structures. Both have a Dirac cone and linear electronic dispersion. Both are expected to have the characteristics of massless Dirac fermions that carry charge, but this is only predicted for silicene and has not been observed, likely because it is expected to occur only with free-standing silicene which has not been synthesized. It is believed that the substrate which the silicene is made on has a substantial effect on its electronic properties

Its buckled structure gives silicene a tuneable **band gap** by applying an external electric field.

Figure panels (a) $E_\perp = 0$, (b) $E_\perp = 0.51$ V/Å, (c) $E_t = 1.03$ V/Å — Silicene; (d) $E_\perp = 0$, (e) $E_\perp = 0.51$ V/Å, (f) $E_t = 1.04$ V/Å — Germanene. Horizontal axis: MK/10 ← K → KΓ/10.

Because of its tuneable band gap, scientist try to make prototypes of FETs using silicene. It is widely believed that if they construct an appropriately functioning silicene transistor, it will be incorporated into the current silicone technology easier than graphene.

Silicene FET

Top Gate
SiO₂ Dielectric Region
h-BN Buffer Layer
Source
Drain
h-BN Buffer Layer
SiO₂ Dielectric Region
Bottom Gate

Possible elaborative questions:

1) Drude model
2) Problems with the Drude-model. The Sommerfeld-modell
3) Electrons in a periodic potential, Bloch-theorem
4) Band structure and the classification of solids
5) Intrinsic Semiconductors
6) Doped semiconductors and the Hall-effect
7) p-n junctions, diodes
8) p-n junctions and the light
9) Tunnel diode
10) Transistors, bipolar junction transistor
11) MOSFETs
12) Superconductivity, behaviour in magnetic field
13) Theory and applications of Superconductivity

Sample test questions to practice (true or false)

1. **The conductivity of metals is increasing with temperature, because the electrons will have more kinetic energy and thus higher velocity.**

2. **Insulators don't conduct electricity because high temperature generates too much lattice defects.**

3. **The band theory of solids can be explained by quantum physics but not by classical physics.**

4. In n type semiconductors the majority carriers are the electrons, but the concentration of holes are still nonzero.

5. In tunnel-diodes, electrons can cross the junction only by tunnelling.