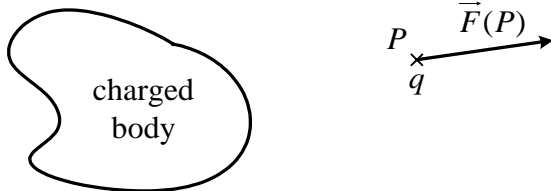


Electricity

The Electric Field Vector

Many phenomena are most conveniently described in terms of the field concept. Any region in space in which an electric charge experiences a force is called an electric field. Consider a charged body at rest in our reference system. This charged body sets up an electric field in space.



Put a point charge q into the point P and measure the force acting on it. We find that the force acting on the point charge q is proportional to its charge. The ratio of the two physical quantities is independent of the point charge and depends only on the field:

$$\vec{E}(P) = \frac{\vec{F}(P)}{q}$$

By definition the intensity of the electric field numerically equals the force acting on a unit point charge at a given point of the field. The direction of the vector \vec{E} coincides with that of the force acting on a positive charge.

Unit of the electric field:

$$[E] = 1 \frac{N}{C}$$

When several charges are present the resultant electric field at any point is the vector sum of the electric fields produced at the point by each charge. This statement expresses the principle of superposition or independent action.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n = \sum_{i=1}^n \vec{E}_i$$

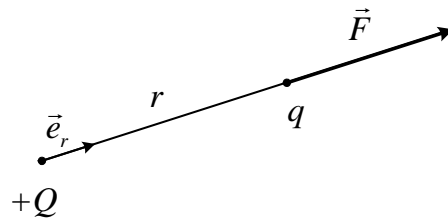
Coulomb's law, Force between Point Charges

In 1785 Coulomb studied the field created by a point charge. A point charge is defined as a charged body whose dimensions may be disregarded in comparison with the distances from this body to the other bodies carrying electric charge.

Coulomb's Law states that a point charge Q at rest sets up a central and conservative electric field around itself. The centre of the field is the point charge. In this electric field the force acting on a point charge q is proportional to the magnitude of the two point charges and inversely proportional to the square of the distance between them. The direction of the force coincides with the straight line connecting the charges.

$$\vec{F} = k \frac{qQ}{r^2} \vec{e}_r$$

where \vec{e}_r is the unit vector directed from Q to q , and $k > 0$ constant.



The value of the proportionality constant depends only on the choice of the charge unit. If the unit of charge is: 1 coulomb = 1C

$$k = 9 \cdot 10^9 \frac{Nm^2}{C^2}$$

One coulomb is the magnitude of the point charge if the force between two identical point charges equals $9 \cdot 10^9 N$ when they are 1 m apart.

The base unit of the electricity is the unit of the electric current. It means that the unit of charge is a derived unit. A coulomb is defined as the amount of charge that flows through any cross section of a wire in 1 second if there is a steady current of 1 Ampere in the wire.
 $1C = 1As$

The proportionality factor:

$$k = \frac{1}{4\pi \epsilon_0}$$

ϵ_0 is called the permittivity constant of vacuum.

$$\epsilon_0 = 8,854 \cdot 10^{-12} \frac{C^2}{Nm^2}$$

The voltage and the first law of electrostatic field

The field due to a point charge at rest is conservative and the principle of superposition is valid for electric fields. That is, the electric field due to any charge distribution, at rest in our inertial system, is a conservative field. This field is called electrostatic field.

The work done by the field on a point charge as it moves from point 1 to point 2 is:

$$W_{1,2} = \int_1^2 \vec{F} \cdot d\vec{r} = \int_1^2 q\vec{E} \cdot d\vec{r} = q \int_1^2 \vec{E} \cdot d\vec{r}$$

The **voltage** between two points:

$$U_{1,2} = \frac{W_{1,2}}{q} = \int_1^2 \vec{E} \cdot d\vec{r}$$

The voltage or potential difference depends only on the field and the two points but is independent of the charge. The potential difference or voltage between two points in an electric field is the work done by the field on a unit positive charge as it moves between the points.

The unit of potential difference:

$$[U] = \frac{1J}{1C} = 1 \text{ volt} = 1V$$

This physical quantity U (sometimes also denoted by V) is called potential. The potential is equal numerically to the potential energy of a unit positive test charge.

In an electrostatic field the potential difference depends only on the initial and final points. According to the agreement the potential is usually chosen to be zero at the infinity. In this case $U(\infty) = 0$ and one can write

$$U(P) = \int_P^{\infty} \vec{E} \cdot d\vec{r}$$

The potential numerically equals the work done by the field on a unit positive charge when it is moved from the point P to infinity.

The conservativity of the electrostatic field means that the work done on any closed path by the field is zero – that is, the potential difference is also zero. This is the integrated form of the first law of electrostatics.

$$\oint_g \vec{E} \cdot d\vec{r} = 0$$

The circle on the integral sign indicates that integration is performed along a closed contour.

The differential form of the first law of electrostatics:

$$\nabla \times \vec{E} = 0$$

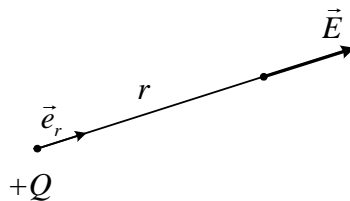
As the force related to the potential energy in conservative field:

$$\vec{F} = -\text{grad}V \quad /: q$$

$$\vec{E} = -\text{grad}U$$

The intensity of the electric field is the negative gradient of the potential.

Electric field and potential due to a point charge



$$\vec{E} = \frac{\vec{F}}{q} \quad \vec{F} = k \frac{qQ}{r^2} \vec{e}_r$$

It is easy to obtain the field:

$$\vec{E} = k \frac{Q}{r^2} \vec{e}_r$$

Now let us calculate the potential.

$$-dU = \vec{E} \cdot d\vec{r} = k \frac{Q}{r^2} \vec{e}_r \cdot d\vec{r}$$

$$-\int_r^{\infty} dU = \int_r^{\infty} k \frac{Q}{r_*^2} dr_*$$

$$U(r) - U(\infty) = kQ \left[-\frac{1}{r_*} \right]_r^{\infty}$$

Thus the potential:

$$U(r) = k \frac{Q}{r}$$

Gauss's Law

$$\oint_A \vec{D} \cdot d\vec{A} = Q$$

It is easy to realize that this connection is valid for any closed surface if Q is enclosed. Moreover, it is valid in the presence of any substance. The above equation is called Gauss's Law or the Second Law of Electrostatics.

The flux of the electric induction through a closed surface equals the algebraic sum of the charges enclosed by this surface.

Positive charges are the sources of the field

Negative charges are the sinks of the field.

Current density and Current

First we introduce the current density vector \vec{J} . The magnitude of the current density vector numerically equals the net charge flowing through a unit perpendicular (to the flow) area in unit time. The direction of \vec{J} is taken as the velocity of the ordered motion of the positive charge carriers (or opposite to the velocity of the ordered motion of the negative charge carriers).

Unit of the current density:

$$[J] = 1 \frac{C}{m^2 s} = 1 \frac{A}{m^2}$$

The current density vector consists of two parts:

$$\vec{J} = \rho\vec{v} + \vec{j}$$

$\rho\vec{v}$: convection current density is associated with beams of charged particles or motion of charged insulator, ρ is the volume charge density and \vec{v} is its velocity

\vec{j} : conduction current density

The direction of the current density is opposite that of the electron average drift speed. If we know the current density vector at each point along a surface A, we can calculate the electric current passing through this surface:

$$I = \int_A \vec{J} d\vec{A}$$

Current is a scalar algebraic quantity. The sign of the current is determined by the choice of the surface normal vector \vec{A} . The current through an area is the net charge flowing through the area in unit time. This is the rate of flow of charge.

$$I = \frac{dQ}{dt}$$

The charge Q that passes a given cross-section in a given time interval is given by:

$$Q = \int_{t_1}^{t_2} I dt$$

Units:

$$[I] = 1 \frac{C}{s} = 1A$$

$$[J] = 1 \frac{A}{m^2}$$

Electromotive Force (EMF)

If only an electric field acts on the charge carriers, the charge flows through the wire from the higher potential to the lower potential (positive carriers) and this would lead to the equalization of potentials and stopping of the current.

To maintain a current for sufficiently long time it is necessary to have the aid of forces of non-electrostatic origin, called extraneous forces. The extraneous forces move the positive charge carriers from the lower potential back to the higher potential. These extraneous forces may be of chemical nature in a battery or cell, or magnetic nature in an alternating-current generator. Extraneous force appears as the magnetic field varies with time. (This will be discussed later.) The device in which extraneous forces act to maintain the potential difference is called a seat of electromotive force. It has two terminals, the higher potential is called the positive terminal and the lower potential is called the negative terminal.

The symbol of the extraneous force is: \vec{F}^* .

The strength of the extraneous field is:

$$\vec{E}^* = \frac{\vec{F}^*}{q}$$

The electromotive force (emf) is defined as the work done by the extraneous forces on a unit positive charge as it moves between the terminals:

$$\mathcal{E}_{+-} = \frac{W_{+-}^*}{q} = \int_{-}^{+} \vec{E} \cdot d\vec{r}$$

Unit:

$$[\mathcal{E}] = 1 \frac{J}{C} = 1V = 1 \text{ volt}$$

We suppose that the emf is independent of the path taken within the seat of emf.

The integral form of Ohm's Law

Ohm's Law was discovered experimentally: the current passing through a homogeneous conductor is proportional to the potential difference across its terminals. Their ratio is called the electric resistance of the conductor R.

$$R = \frac{U}{I}$$

Unit of resistance:

$$[R] = 1 \frac{V}{A} = 1\Omega = 1 \text{ ohm}$$

Resistance as the function of geometrical sizes

Consider a homogeneous cylindrical conductor of length l , cross-sectional area A . The increase of the length of the conductor by a factor of two is the same as the connection of another conductor in series to the original one, so the resistance is also increased by a factor of two. Therefore the resistance is proportional to the length of the wire. The increase of the cross-section of the conductor by a factor of two is the same as the connection of another conductor in parallel, so in this case the resistance is decreased by half. Therefore the resistance is inversely proportional to the cross-section of the conductor.

$$R = \rho \frac{l}{A}$$

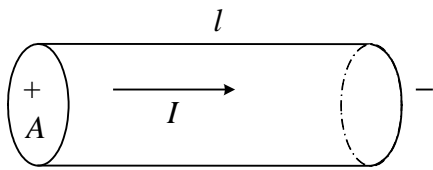
The proportionality factor ρ is termed the resistivity of the substance, its unit is Ωm or $\frac{\Omega mm^2}{m}$.

The differential Ohm's Law

The electric resistance of the conductor R can be expressed by the electric field and current density vectors inside the conductor.

$$R = \frac{U}{I} = \frac{\int_{+}^{-} \vec{E} \cdot d\vec{r}}{\int_{A} \vec{j} \cdot d\vec{A}}$$

In a homogeneous cylindrical conductor of length l , cross-sectional area A , and resistivity ρ it is very simple:



$$U = \int_{+}^{-} \vec{E} \cdot d\vec{r} = \int_{+}^{-} E dr = E \int_{+}^{-} dr = El$$

and

$$I = \int_{A} \vec{j} \cdot d\vec{A} = \int j dA = jA,$$

therefore

$$R = \frac{U}{I} = \frac{El}{jA}$$

On the other hand the resistance is proportional to the length and inversely proportional to the cross-section of the conductor:

$$R = \rho \frac{l}{A}$$

From these equations we can find that $E = \rho j$. The electric field and current density vectors remain parallel in the isotropic case and we can write:

$$\vec{E} = \rho \vec{j}$$

This equation is the differential form of Ohm's Law. Introducing the electrical conductivity σ (sometimes denoted by γ) as the reciprocal value of the resistivity of the substance:

$$\rho = \frac{1}{\sigma}$$

we obtain another form of the differential Ohm's Law:

$$\boxed{\vec{j} = \sigma \vec{E}}$$

In the case of metals, with increasing temperature the conductivity decreases, while for semiconductors it increases.

The differential Ohm's Law is a strict proportionality between \vec{j} and \vec{E} , but it is not always valid, because in some cases σ and \vec{j} are not independent. However, for usual metals, Ohm's Law is a very good approximation, thus this is one of the most important formula in electricity.

The resistance of a large group of metals and alloys vanishes in jump at a temperature close to the absolute zero. This is called superconductivity, and was discovered by Kammerling and Onnes (1911). For example, in the case of lead $T_{cr} = 7K$.

Work and power in stationary current circuit

Consider a consumer in a circuit, having a current I and potential difference U_{12} between the two terminals. As charge passes through this circuit element the electric field does work on the charge. In time t the amount of charge passes through the consumer, in case of stationary current is $Q = It$, so the work is:

$$W = QU_{12} = U_{12}It$$

By means of this work the electric field transfers energy into this portion of the circuit. The rate of the energy transfer is the power:

$$P = \frac{W}{t} = U_{12}I$$

In case of pure resistance:

$$U_{12} = IR$$

$$P = U_{12}I = I^2R = \frac{U_{12}^2}{R}$$

This is the power input to the resistor. The circulating charges give up energy to the atoms of the resistor when they collide with them and the temperature of the resistor increases.

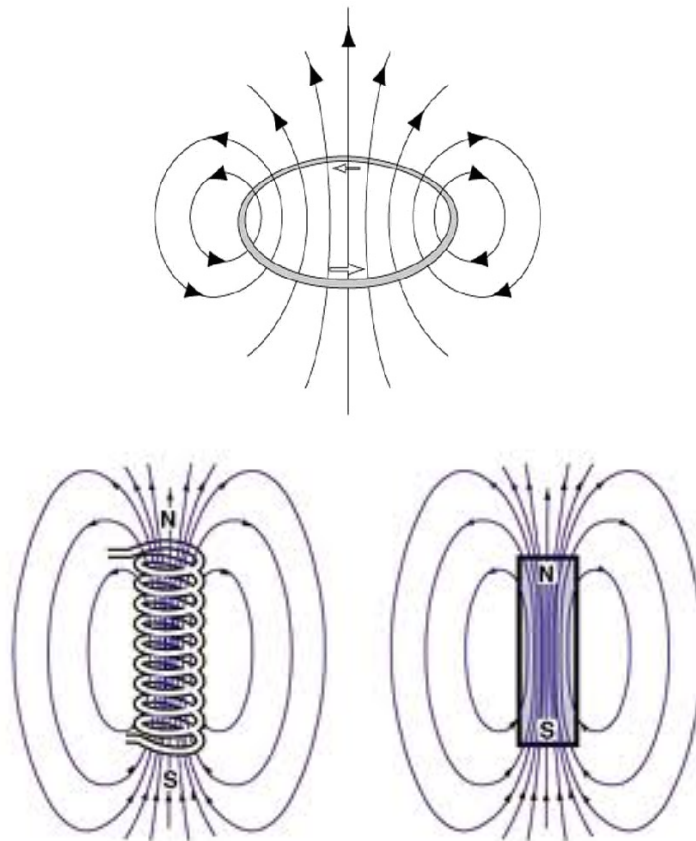
We say that energy is dissipated in the resistor at a rate IR^2 .

Electromagnetism

Ampere's law: The magnetic field in space around an electric current is proportional to the electric current which serves as its source

$$\oint_g \vec{H} d\vec{s} = \sum_j I_j$$

The magnetic field of a current loop, a solenoid and a bar magnet:



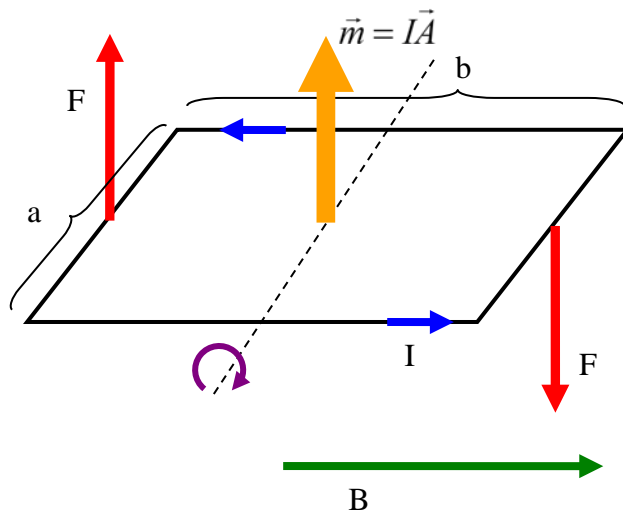
Lorentz-force: The force acting on a particle of electric charge q with instantaneous velocity v , due to an external magnetic field B , is given by

$$\vec{F} = q \vec{v} \times \vec{B}$$

Ampère-force: When a wire carrying an electric current is placed in a magnetic field, each of the moving charges, which comprise the current, experiences the Lorentz force, and together they can create a macroscopic, so called Ampere-force on the wire (sometimes called the Laplace force). By combining the Lorentz force law above with the definition of electric current, the following equation results, in the case of a straight, stationary wire:

$$\vec{F} = I \vec{\ell} \times \vec{B}$$

Now examine the forces acting on the sides of a current-loop:



In homogeneous magnetic field the net force is zero: $\sum \vec{F} = 0$

However, there is a **torque**:

$$2 \cdot BIa \cdot \frac{b}{2} = BIab = BIA,$$

where $b/2$ is the lever arm. Generally:

$$\vec{\tau} = I\vec{A} \times \vec{B} = \vec{m} \times \vec{B},$$

where

$$\boxed{\vec{m} = I\vec{A}}$$

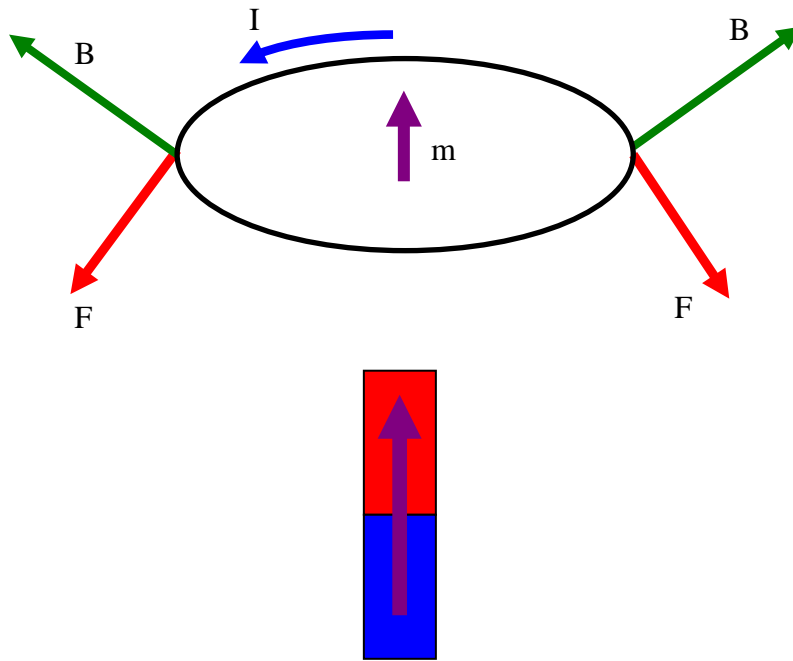
is the **magnetic dipole moment**. The formula for the torque is valid for every kind of magnetic moment. Because of this torque, the moments tend to rotate towards the direction of the magnetic field.

The potential energy of the moment in a field B :

$$\boxed{E_p = -\vec{m} \cdot \vec{B}}$$

When the moment points to the direction of the field, this is always the energy-minimum.

Force between magnets and current-loops:



We can see that opposite poles attract each other.

Magnetic Gauss-law: The magnetic flux on a closed surface is zero.

$$\oint_A \vec{B} d\vec{A} = 0$$

The magnetization \vec{M} is the net magnetic moment in a unite volume:

$$\vec{M} = \lim_{\Delta V \rightarrow 0} \frac{\Delta \vec{m}}{\Delta V}$$

Normally thermal motion causes the magnetic moments to become randomly oriented, thus

$$\vec{H} = 0 \Rightarrow \vec{M} = 0$$

If one applies an external field H , a torque will be exerted on the moments. Stronger the external magnetic field, more the moments become orientated, thus higher the magnetization will be.

The usual linear approximation:

$$\boxed{\vec{M} = \chi \vec{H}}$$

where χ is the magnetic susceptibility.

Now the magnetic induction is, by definition: $\vec{B} = \mu_0 (\vec{H} + \vec{M})$. Substituting the linear approximation $\vec{M} = \chi \vec{H}$ into this definition we obtain:

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \chi \vec{H}) = \mu_0 (1 + \chi) \vec{H} = \mu_0 \mu_r \vec{H} ,$$

where $\mu_r = 1 + \chi$ is the relative permeability, $\mu = \mu_0 \mu_r$ is the absolute permeability.

Thus we have a material/constitutive relation:

$$\boxed{\vec{B} = \mu_0 \mu_r \vec{H}}$$

or simply $\vec{B} = \mu \vec{H}$. We must emphasize that it is just an approximation.

The energy density of the magnetic field:

$$w_m = \frac{1}{2} \vec{B} \cdot \vec{H}$$

The total magnetic energy in a V volume

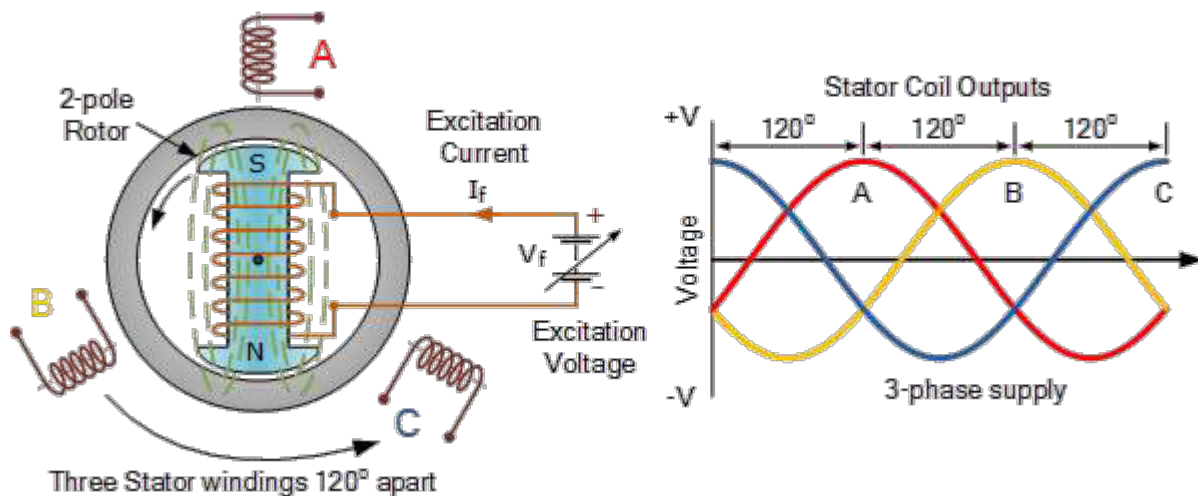
$$W = \int_V w_m dV = \frac{1}{2} \int_V \vec{B} \cdot \vec{H} dV$$

Electromagnetic induction:

Electromagnetic induction is the production of an electromotive force across a conductor when it is exposed to a varying magnetic field. It is described mathematically by Faraday's law of induction

$$\oint \vec{E} d\vec{r} = - \frac{d}{dt} \int \vec{B} d\vec{A} \quad \text{or briefly} \quad \mathcal{E} = - \frac{\Delta \Phi}{\Delta t}$$

Most electricity is produced at electric power plants by electromagnetic induction, when a magnet is rotating next to some coils.



The Maxwell-equations

Maxwell equations are at the center of the theory of electromagnetism. The first part of the first law tells us that an electric current gives rise to a magnetic field that circles the wire. The second part was introduced by Maxwell himself and says that a changing electric field (the “displacement current” term) also gives rise to a magnetic field that encircles the E-field.

$$\oint \vec{H} d\vec{r} = \sum_k I_k + \frac{d}{dt} \int \vec{D} d\vec{A} \quad ,$$

The other equations are just the Faraday's law of induction, and the electric and magnetic Gauss' law:

$$\oint \vec{E} d\vec{r} = -\frac{d}{dt} \int \vec{B} d\vec{A}$$

$$\oint_A \vec{D} \cdot d\vec{A} = Q$$

$$\oint_A \vec{B} d\vec{A} = 0$$

We can solve these equations only if we have some kind of material relations, like

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} \quad \text{and} \quad \vec{B} = \mu_0 \mu_r \vec{H}$$

Look now at the first law. This says that a changing electric field gives rise to a circling magnetic field. The second law states that this magnetic field itself will produce a changing electric field which will also give rise to a changing magnetic field and so on.

This phenomenon is known as the propagation of electromagnetic waves, for example when sunlight travel through a vacuum without any medium. We examine EM waves in the next chapter.

Electromagnetic waves

The existence and some of the properties of the electromagnetic waves can be derived from the Maxwell equations. For example that all propagating EM waves travel at a single speed, the speed of light. And this speed can be directly determined from the constants found within Maxwell's Equations and the material relations.

In the EM wave, the electric and the magnetic field can have a plane-wave form:

$$E = E_0 \sin \left[2\pi \left(ft - \frac{x}{\lambda} \right) \right] \quad \text{and} \quad H = H_0 \sin \left[2\pi \left(ft - \frac{x}{\lambda} \right) \right],$$

where f is the frequency ($f=1/T$), λ is the wavelength and

$$\boxed{c = f \lambda}$$

is the velocity of light. Moreover, it is easy to see that $c = \frac{1}{\sqrt{\epsilon\mu}}$, thus the speed of light

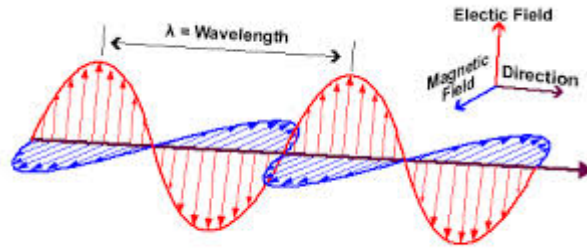
in a medium depends on the relative permittivity and permeability of the medium.

We can write the formula into an equivalent form:

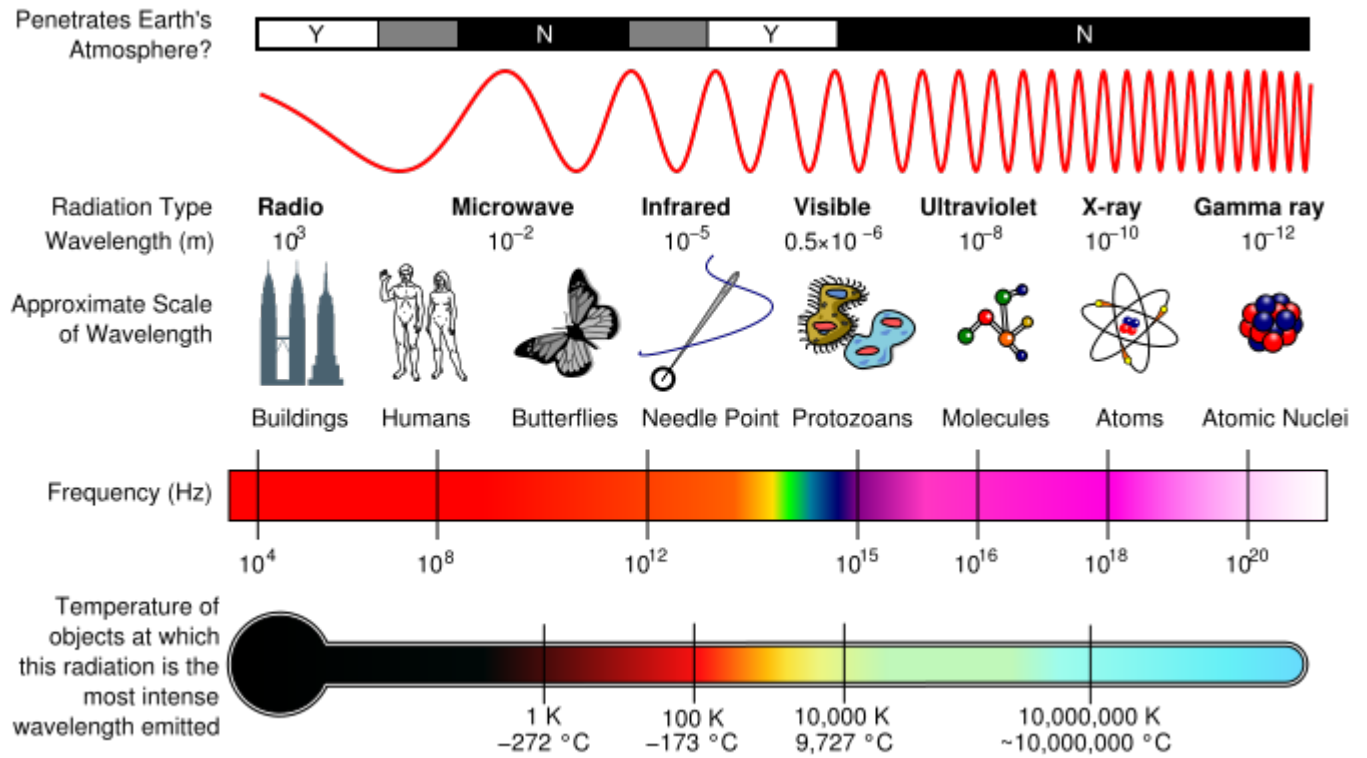
$$E = E_0 \sin(\omega t - kx),$$

where $\omega(=2\pi f)$ is the angular frequency, k is the (angular) **wavenumber**: $k = \frac{2\pi}{\lambda}$,

For transverse waves the wavering quantity (e.g. displacement of the medium) is perpendicular to the direction of propagation of the wave, while in longitudinal waves it is parallel to the propagation of the wave. EM radiation are **transverse** waves.



The Electromagnetic Spectrum with Radiation Types



Not only visible light, but X-rays, radio waves, television waves, Wi-Fi signals, Bluetooth signals, cell phone transmission and GPS signs are all electromagnetic waves.

Magnetic properties of matter

The magnetic behaviour of the materials are determined mostly by the electrons of the material. The electrons provide two sources of magnetic moment:

1. Moments belongs to the orbital motion of the electron. It has different magnitude for different electrons.
2. Moments associated with the spin of the electron. The magnitude of this moment is fixed, but the direction can change, of course.

If we place a typical material into the magnetic field, there are two, qualitatively different processes:

A) **Diamagnetic process:** Because of the change of the magnetic field, an electromotive force is induced, which alters the orbital velocity of electrons around their nuclei, thus changing the magnetic dipole moment in the direction. The induced magnetic moment and

magnetization \vec{M} will oppose the external field \vec{H} , because of Lenz law. This process is quite weak, but present in every material.

B) Paramagnetic process: Some atoms and molecules have magnetic moments without any external field. Usually the atomic orbitals of these substances are partially filled and hence there exists unpaired electrons with uncompensated (spin and/or orbital) magnetic moments. The magnetic field exerts a torque onto these moments thus they change their direction towards the field and therefore \vec{M} will have the same direction as \vec{H} . This effect is usually slightly stronger than the diamagnetic process.

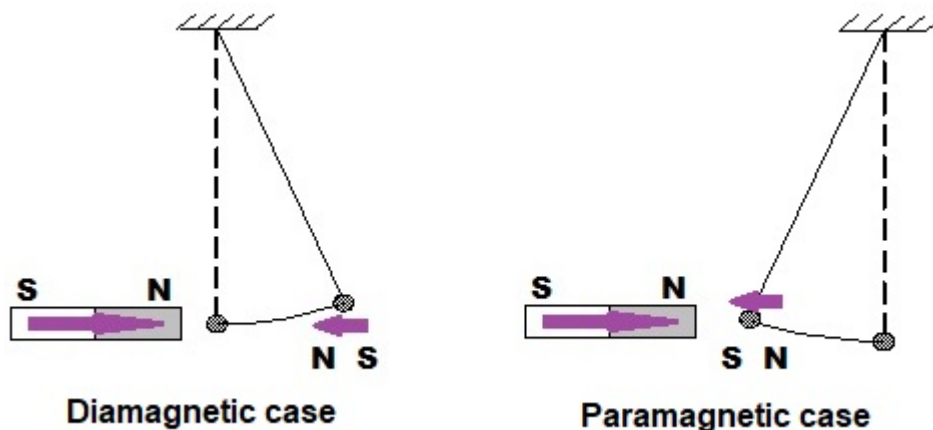
Diamagnetism

Diamagnetism is the property of an atom or molecule where the diamagnetic effect is the dominant. The susceptibility is negative:

$$\chi < 0 \text{ (e.g. } \chi = -10^{-5} \text{)} \quad \text{and} \quad \mu_r < 1 \text{ (e.g. } \mu_r = 0.999992 \text{ for water)}$$

This effect causes a repulsion of the atom away from the magnet. Consequently, diamagnetism is a form of magnetism that occurs only in the presence of an externally applied magnetic field. It is generally a quite weak effect in most materials, although superconductors exhibit a similar, but strong effect.

Examples of diamagnetic materials are copper, lead, gold, silver, water, noble gases (like argon) etc.



The interaction of a bar-magnet and a dia/paramagnetic light ball (repulsion/attraction)

Paramagnetism

In paramagnetic materials, the paramagnetic process is stronger than the diamagnetic one. These materials have a relative magnetic permeability greater than one (or, equivalently, a positive magnetic susceptibility).

$$\chi > 0 \text{ (e.g. } \chi = 10^{-4} \text{)} \quad \text{and} \quad \mu_r < 1 \text{ (e.g. } \mu_r = 1.0001 \text{)}$$

Hence paramagnets are attracted to magnetic poles. Some examples include aluminium, chromium, manganese, oxygen, platinum, alkali and alkaline earth metals.

The magnetisation induced by the applied field is approximately linear in the field strength and rather weak. It typically requires a sensitive analytical balance to detect the effect. Paramagnetism is a form of magnetism which occurs only in the presence of an externally applied magnetic field. Unlike ferromagnets, paramagnets do not retain any magnetization in the absence of an externally applied magnetic field, because thermal motion causes the spins to become **randomly** oriented without it. Thus the moments cancel out one another,

the total magnetization will drop to zero when the applied field is removed. Even in the presence of the field there is only a small induced magnetization because only a small fraction of the moments will be oriented into the direction of the field. This fraction is proportional to the field strength and this explains the linear dependency. Thermal random motion decreases the effect of the field, thus

$$\chi \sim \frac{1}{T}$$

This is the Curie-law.

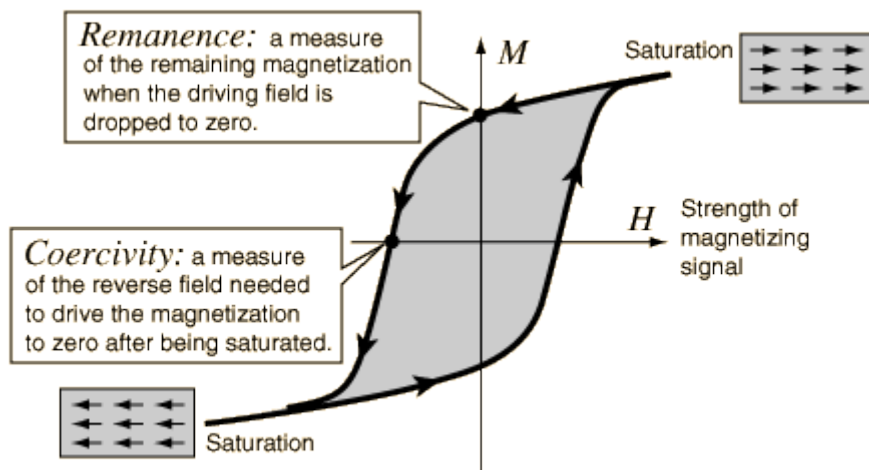
Ferromagnetism

The attraction experienced by ferromagnets is non-linear and much stronger, so that it is easily observed, for instance, in the case of a bar-magnet.

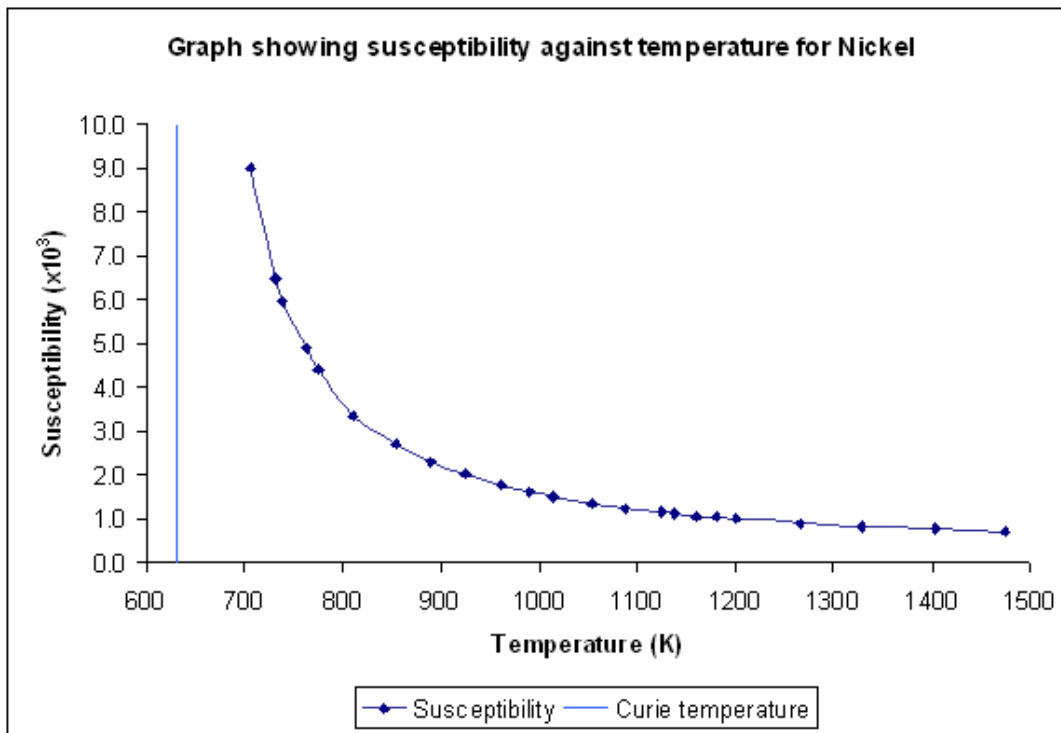
$$\chi \gg 1 \quad \mu_r \gg 1$$

the tiny magnetic fields add up and create a strong magnetic field

When a ferromagnetic material is magnetized in one direction, it will not relax back to zero magnetization when the imposed magnetizing field is removed. The amount of magnetization it retains at zero driving field is called its **remanence**. It must be driven back to zero by a field in the opposite direction; the amount of reverse driving field required to demagnetize it is called its **coercivity**. If an alternating magnetic field is applied to the material, its magnetization will trace out a loop called a **hysteresis loop**.



Ferromagnetism is an ordered state, therefore it cannot survive at high temperatures. For each ferromagnetic material there is a critical (so called Curie) temperature T_C when a ferromagnetic –paramagnetic phase transition occurs.



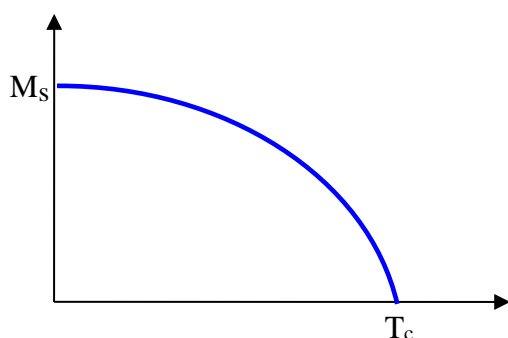
A Curie-Weiss Law: Valid above T_C .

$$\chi \sim \frac{1}{T - T_C}$$

Domains: A magnetic domain is a region within a magnetic material in which the magnetization is in a uniform direction. This means that the individual magnetic moments of the atoms are aligned with one another and they point in the same direction.



This ordered alignment is perfect only at $T=0$. Above the absolute zero, the moments have some random motion, thus the magnetization cannot reach saturation.



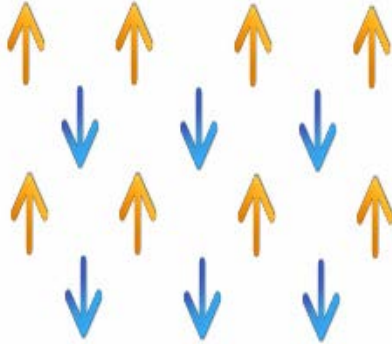
Exchange or Heisenberg interaction:

In some materials, there is a quantum-mechanical interaction between the magnetic moments:

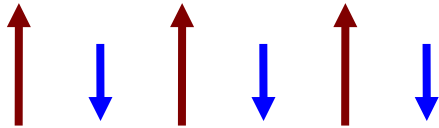
$$E = -J \sum \vec{m}_i \vec{m}_j$$

If $J > 0$, the exchange energy favours electrons with parallel spins; this is a primary cause of ferromagnetism in materials.

If $J < 0$, the opposite spin-configuration will be the energy-minimum. It is called antiferromagnetism.



Sometimes there are two sublattices with different total magnetic moment. If $J < 0$, locally this material looks like antiferromagnetic, but there is a significant net magnetization, thus it behaves like a ferromagnetic material in experiments. We call it ferrimagnetism, and the most important example are the ferrites.

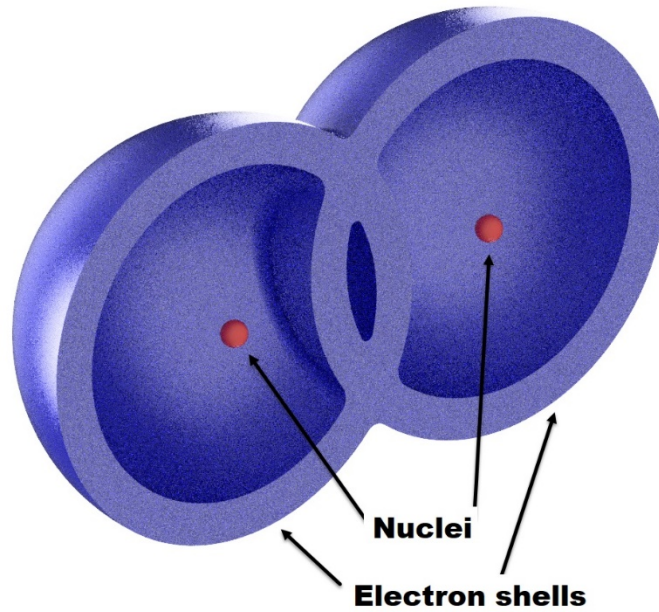


The macroscopic behaviour of the ferrimagnetic materials is very similar to that of ferromagnetic materials. However, microscopically they are very different due to the different sign of J .

Although these consequences of the exchange interaction are magnetic in nature, the cause is not. Indeed, in general, the direct magnetic dipole-dipole interaction between a pair of atoms (due to magnetic moments of their electrons) is negligibly small compared to this interaction.

Types of Exchange:

- a) **Direct Exchange:** it is due primarily to electric repulsion and the Pauli Exclusion Principle. Acts only if the electron-clouds of the atoms touch each other: short range, 1-2 atomic size.



- b) **RKKY** (Ruderman-Kittel-Kasuya-Yosida) **interaction** mediated by conduction electrons: indirect, long range, 15-20 atomic size.
- c) **Superexchange** indirect, mediated by localized electrons.

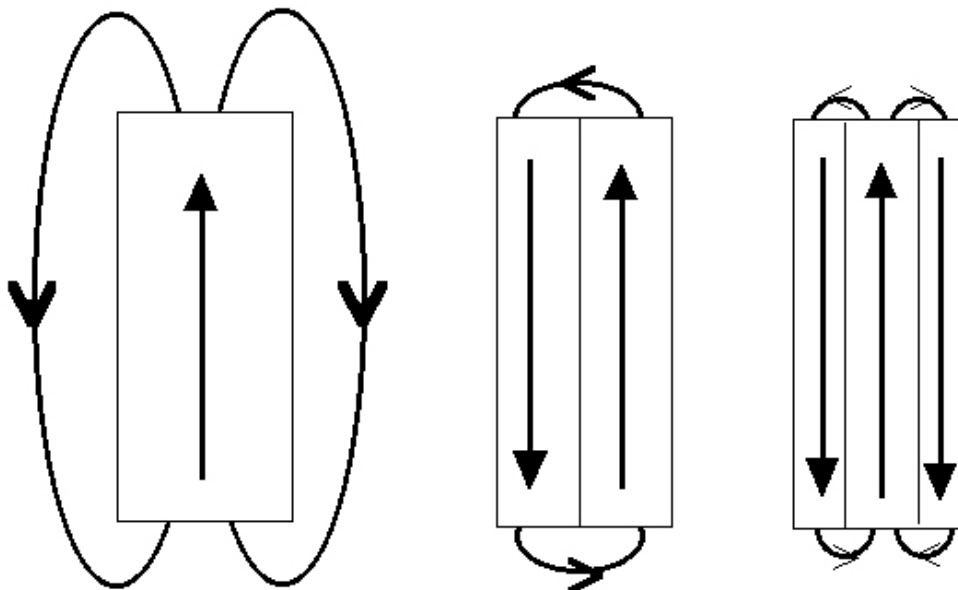
Magnetostatic energy (**Energy density of the magnetic field**):

$$w_m = \frac{1}{2} \vec{B} \cdot \vec{H}$$

Origin: classical electrodynamic.

Consequence. The energy of the magnetic field increases with the 3rd power of the diameter of the sample, thus large single domains are far from the energy minimum.

Domain Formation



Single Domain

Multidomain

Total Energy = Magnetostatic Energy + Wall Energy

Zeeman energy:

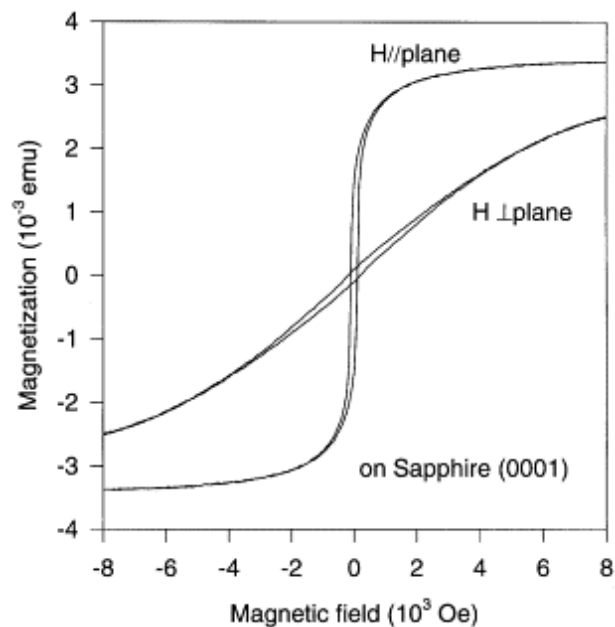
$$E_p = -\vec{m} \cdot \vec{B}$$

Origin: classical electrodynamic, the torque discussed above.

Consequence: the para-, ferro or ferrimagnetic material reacts to the external magnetic field. In a paramagnet, when the field is changing, the individual moments rotate independently. In a ferro- or ferrimagnet, this is not the case. Instead, the domain-walls move to reduce the Zeeman-energy.

Anisotropy:

The directions are not equivalent. For example, to magnetize a ferromagnetic plane is easy if the direction of H is inside the plane and hard if it is perpendicular.

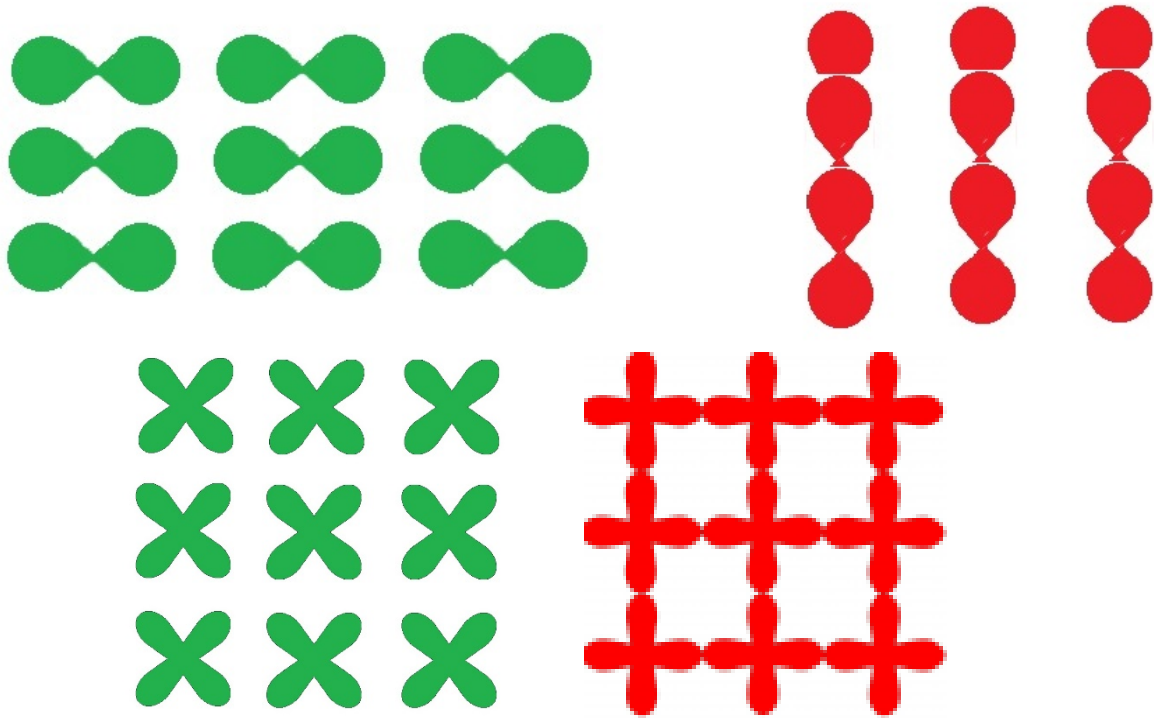


Types of anisotropy:

a) Magnetocrystalline anisotropy. Some crystal-axes are energetically favoured by the orbital and spin moments than others.

Mechanism responsible for the crystal-anisotropy:

- The orbits of the localized electrons (electron-clouds) repel each other, thus if they are not spherical, they do not want to point towards each other. In other words, the orbital motion of the electrons couples with crystal electric field. The orientation of the orbits are in direct connection with the orientation of the magnetic moments.
- The spin-orbit interaction (relativistic quantum mechanics) makes a connection between the orientation of the orbital moment and the spin. So this is the primary source of magnetocrystalline anisotropy.



Thus the energy of the system depends on the direction of the magnetization, i.e. the angles between the moments and the axes of the coordinate system. The concrete function depends on

- the material (iron or cobalt, etc.) and also
- the crystal structure (α iron is bcc - body centered cubic, γ iron is fcc - face centered cubic).

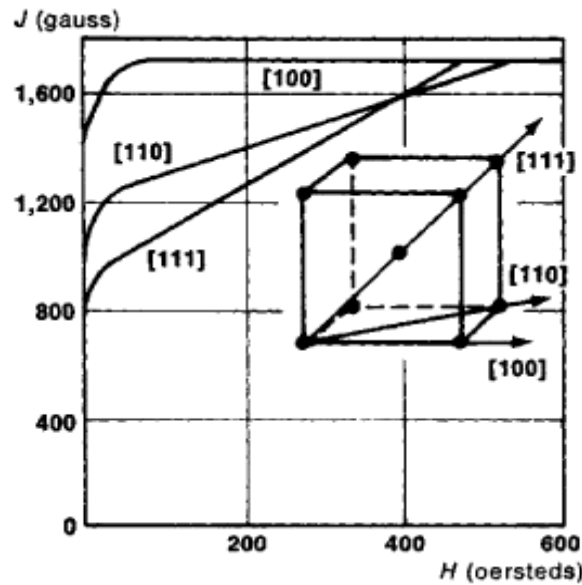
The simplest is the **uniaxial anisotropy**: there is one distinguished axis. If the z axis is taken to be the main symmetry axis of the crystal and φ is the angle between the moments and this z axis:

$$E = K \cdot V \cdot \sin^2 \varphi,$$

Usually $5\text{kJ/m}^3 < K < 10\text{MJ/m}^3$

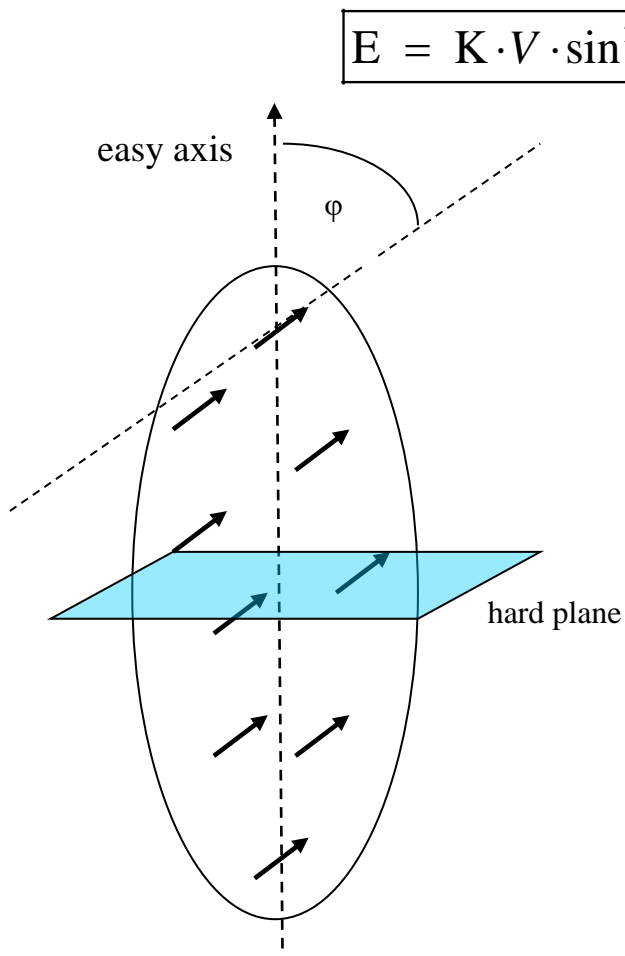
If $K > 0$, then $\varphi = 0$ is the energy-minimum, easy axis, hard plane.

If $K < 0$, then $\varphi = \frac{\pi}{2}$ is the energy-minimum, hard-axis, easy-plane.



b) Shape-anisotropy

The origin of shape anisotropy is classical electrodynamic: the magnetic field and therefore the total amount of the magnetostatic energy depends on the shape of the sample. For example, consider a long thin needle shaped grain. The energy of the magnetic field will be less if the magnetization is along the long axis than if it is along one of the short axes. This produces an easy axis of magnetization along the long axis. In case of elongated samples, the longest axis of the sample is the energetically favoured axis:



A spherical grain, on the other hand, has no shape anisotropy.

c) Stress-anisotropy

Applied or residual stresses can change the magnetic properties of a material. A uniaxial stress can produce a distinguished axis, and it can be the unique easy axis of magnetization if the stress is sufficient to overcome all other anisotropies.

The inverse effect, the so called **magnetostriction** arises from the strain dependence of the anisotropy constants. Upon magnetization, a previously demagnetized crystal experiences a strain that can be measured as a function of applied field along the principal crystallographic axes. A magnetic material will therefore change its dimension when magnetized.

Practical consequence of anisotropy

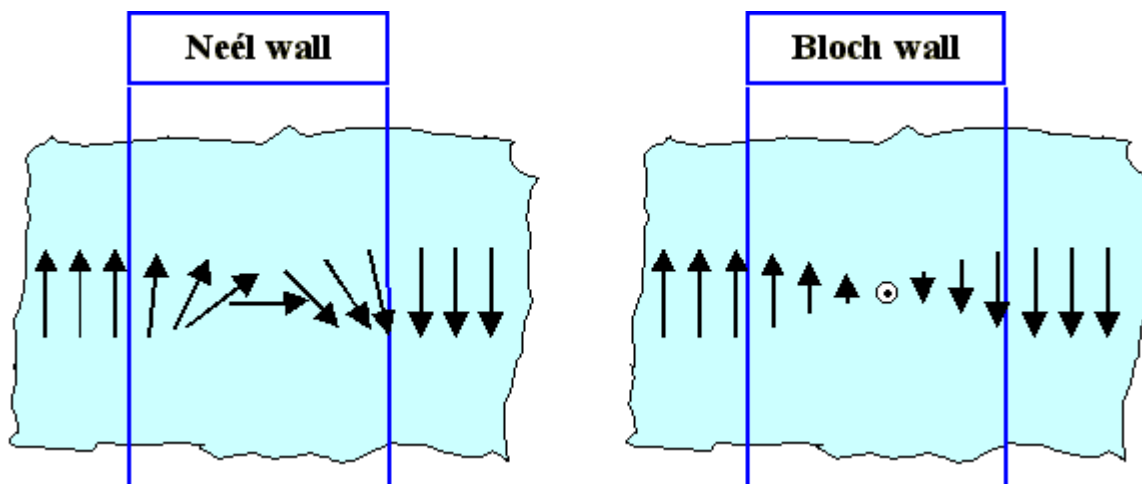
Magnetocrystalline anisotropy has a great influence on industrial uses of ferromagnetic materials. Materials with **high magnetic anisotropy** usually have **high coercivity**; that is they are hard to demagnetize. These are called "hard" ferromagnetic materials, and are used to make permanent magnets. For example, the high anisotropy of rare earth metals (like neodymium) is mainly responsible for the strength of rare earth magnets. During manufacture of magnets, a powerful magnetic field aligns the microcrystalline grains of the metal so their "easy" axes of magnetization all point to the same direction, freezing a high magnetization into the material, thus yielding a strong magnetic field outside.

On the other hand, materials with low magnetic anisotropy usually have low coercivity, their magnetization is easy to change. These are called "soft" ferromagnets, and are used to make magnetic cores for transformers and inductors. The small energy required to turn the direction of magnetization minimizes core losses, energy dissipated in the transformer core when the alternating current changes direction.

Domain walls

In magnetism, a domain wall is an interface separating magnetic domains. It is a transition between different magnetic moments and usually undergoes an angular displacement of e.g. 90° or 180° .

A domain wall is a gradual reorientation of individual moments across a finite distance. The domain wall thickness is usually around 100–150 atoms.



Bloch walls are more typical in macroscopic bulk samples.

The energy of a domain wall is simply the difference between the magnetic moments before and after the domain wall was created. This value is usually expressed as energy per unit wall area.

http://en.wikipedia.org/wiki/Domain_wall_%28magnetism%29

The width of the domain wall varies due to the two opposing energies that create it: the magnetocrystalline anisotropy energy K and the exchange energy J , both of which tend to be as low as possible so as to be in a more favourable energetic state. The anisotropy energy is lowest when the individual magnetic moments are aligned with the crystal lattice axes thus reducing the width of the domain wall close to zero. Thus:

K is high \Rightarrow the wall is thin

Conversely, the exchange energy is reduced when the magnetic moments are aligned parallel to each other and thus makes the wall as thick as possible:

J is large \Rightarrow the wall is thick

In the end a compromise is reached between the two and the domain wall's width is set as such.

In case of Bloch-wall:

$$d_B \sim \sqrt{\frac{J}{K}}$$

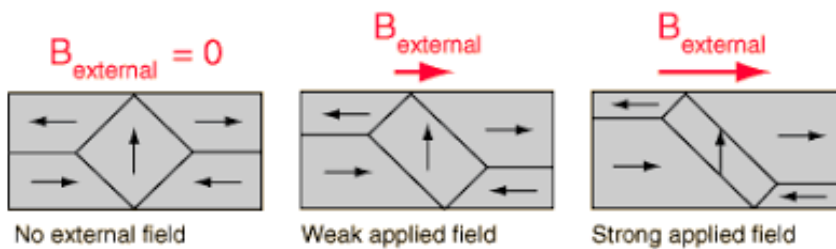
The energy of the (unit area of) domain-wall:

$$w_B \sim \sqrt{JK}$$

Usually $0.1 \frac{mJ}{m^2} \leq w \leq 50 \frac{mJ}{m^2}$

The magnetostatic energy depends on the 3rd power of the diameter, while the energy of the domain-wall depends on the 2nd power only. That is why there are always many domain walls in a macroscopic sample (without external field). On the other hand, if we decrease the diameter, the magnetostatic energy decreases much faster and finally becomes negligible. Under a **critical** diameter walls do not appear, because the energy of that would be larger than the magnetostatic energy and thus it has no sense to appear.

If a weak magnetic field is applied, the individual moments will feel a torque. However, **inside** the domains they will not change their direction, because they are at the bottom of a deep energy well caused by the Heisenberg interaction with their neighbours. However, in the domain walls the external field just tip the balance a little bit and causes a slight rotation of the moments towards the preferred direction. The effective result is the displacement of the domain wall and the increase of energetically favoured domains to the detriment of the other domains.

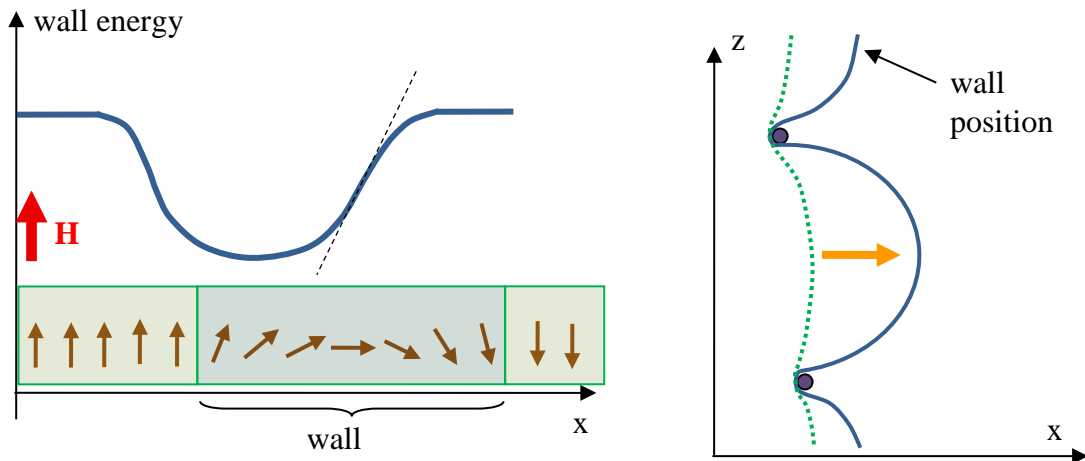


An ideal domain wall would be fully independent of position, but the structures are not ideal and so get stuck on inclusion sites within the medium, also known as crystallographic defects. These include

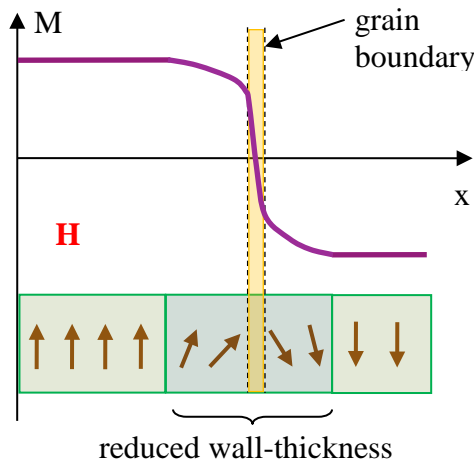
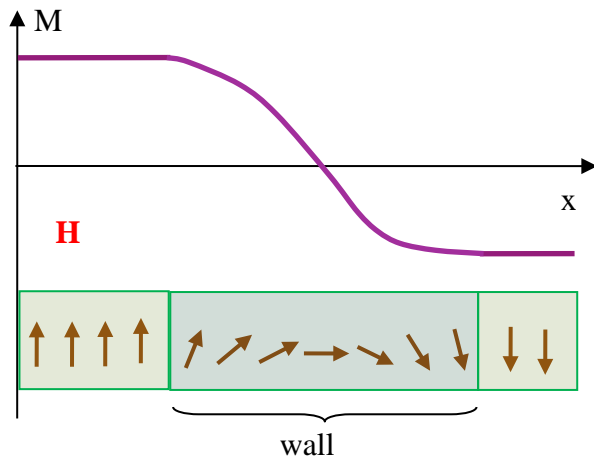
- vacancies (missing atoms),
- interstitial defects
- substitutional defect (different – foreign – atoms)
- dislocations (like edge dislocation)
- grain boundaries.

These defects **inhibit the propagation of the domain walls** through the medium.

This is because the $w_B \sim \sqrt{JK}$ energy of the domain-wall is larger or smaller there, thus they work as repulsive or attractive centers. Consequentially a greater applied magnetic field is required to overcome these sites.



For example, at grain boundaries, the distance between atoms are larger than usual, thus J is smaller. It yields an attractive center for the wall, and the $d_B \sim \sqrt{\frac{J}{K}}$ thickness of the wall is smaller there.



These defects are the main causes of irreversibility and hysteresis losses.

Hysteresis of macroscopic samples

The hysteresis loop shows the irreversible, nonlinear response of a ferromagnet to a magnetic field. It reflects the arrangement of the magnetization in ferromagnetic domains. The magnet cannot be in thermodynamic equilibrium anywhere around the open part of the curve!

The change of the magnetic state of a ferromagnetic body under the effect of a changing applied magnetic field is a complex process, involving reversible and irreversible changes of the magnetization. Due to irreversible magnetization processes, the change of the magnetic state is accompanied by losses, manifested by the presence of magnetic hysteresis. The physical origin of the losses is that a real material has a microstructure, material defects, and internal stresses responsible for the details of the hysteresis loop. The magnetization changes discontinuously in space and in time by jumps from defect to defect, giving rise to magnetic noise. This process and the corresponding noise are named after its discoverer, H. Barkhausen (1919) as **Barkhausen jumps** and **Barkhausen noise**. In case of conductors, the origin of the noise is partly the induced eddy currents, because sudden changes in magnetism induces electric field and therefore electromotive force.

Let us start to examine the concrete mechanism of hysteresis. At the $M=0$ position, the magnetic moments of the individual domains cancel out one another.

There are two magnetization processes for a ferromagnet:

A) **Domain-wall motion.** If the domain walls are perfectly free, the area of the loop will be zero. Every crystal-defect makes the potential energy of the walls space-dependent, thus lead to irreversibility and Barkhausen-jumps.

B) **Magnetization rotation** of an entire domain (collective, coherent rotation). Because of anisotropy, these are usually irreversible and this is the other source of Barkhausen-jumps.

There are 3 parts of the initial (virgin) curve:

1. The walls start to move but still not reaching the first lattice-defects. This reversible section is usually short.
2. Irreversible part with Barkhausen-jumps. This long period finish when the angle between the magnetic moments and the field becomes small and all domains with very unfavourable Zeeman-energy disappears. However, at the end of this section the sample might not be perfectly saturated. This is because the easy-axes of anisotropy in the grains have a random orientation which usually do not coincide with the external field. Therefore the moments can remain in deep or shallow anisotropy potential-wells for rather large field as well.
3. Increasing the field gradually and reversibly force the moments to leave anisotropy potential-wells and turn towards the direction of the field.

Barkhausen Effect

Cilcks can be heard on a loud speaker

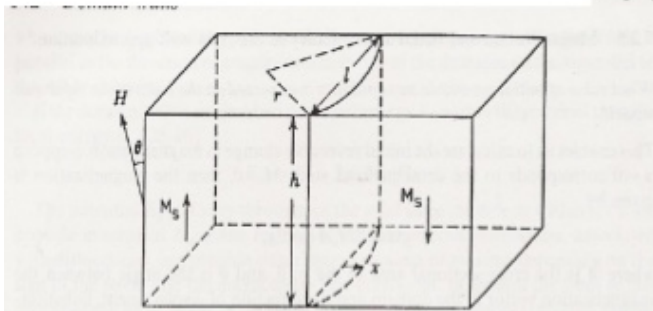
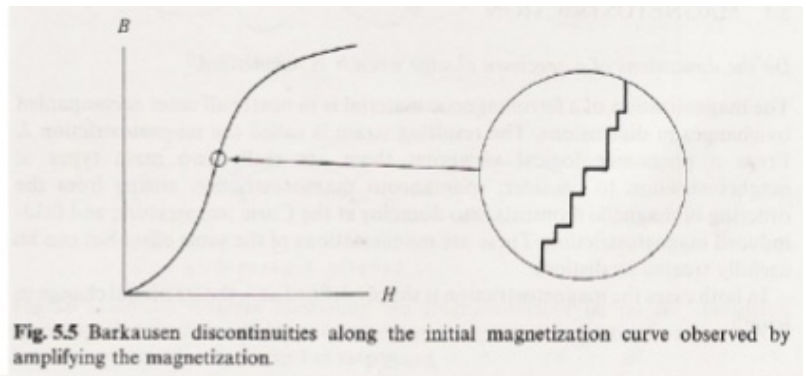
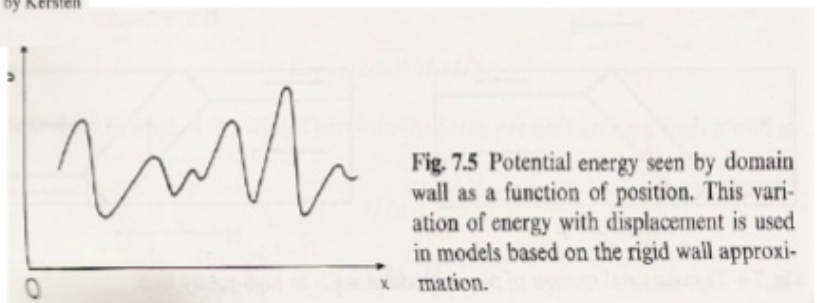


Fig. 7.6 Bending of a domain wall under the action of a field. The domain wall is pinned at the boundaries and expands in the manner of an elastic membrane as described by Kersten and Neel [27,29].

Flexible Domain Wall - Contributes to Reversible M

Domain walls need to cross potential wells in the magnetization process



See e.g. here:

http://www.tf.uni-kiel.de/matwis/amat/elmat_en/kap_4/backbone/r4_3_4.html

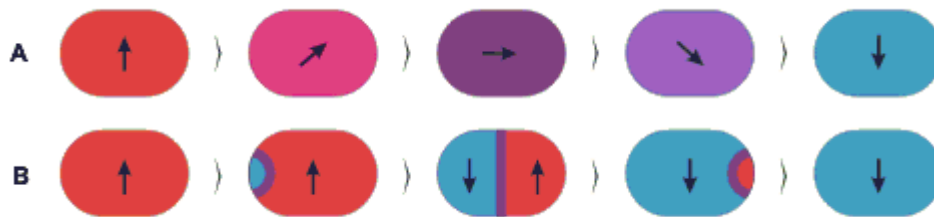
Hysteresis of nanomagnets

We use the term nanomagnet for monodomains, i.e. if there are no domains in the system if $H=0$. This is true if the sample is small enough, typically under 80-100nm diameter, when the surplus energy of the domain wall is larger than the gain in magnetostatic energy.

If the shape-anisotropy is not large, there are two possibilities:

A) Coherent rotation of all moments (usually for smaller samples and for stronger exchange interaction).

B) Nucleation of a second domain, and moving of the new domain wall. (This is general for slightly larger samples and if the exchange interaction is weak compared to the anisotropy).

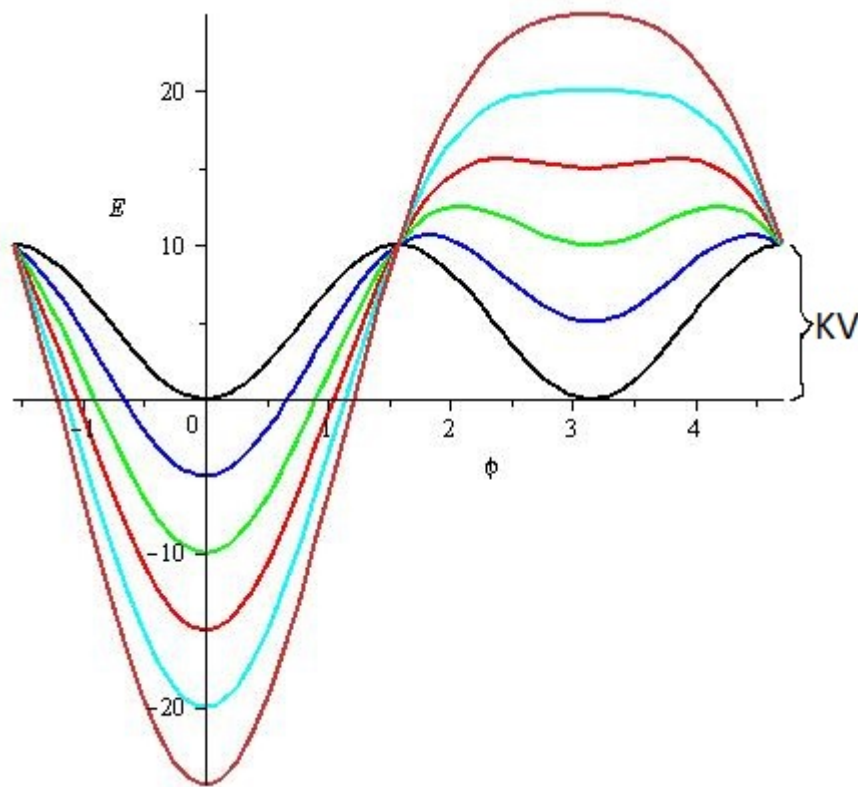


For digital data-recording, none of the above is desirable. We need higher shape-anisotropy, thus we use small needle-shape particles. Because of the small size, the magnetostatic energy is negligible. Because of the coherent rotation, there are no domain walls and the exchange energy is constant.

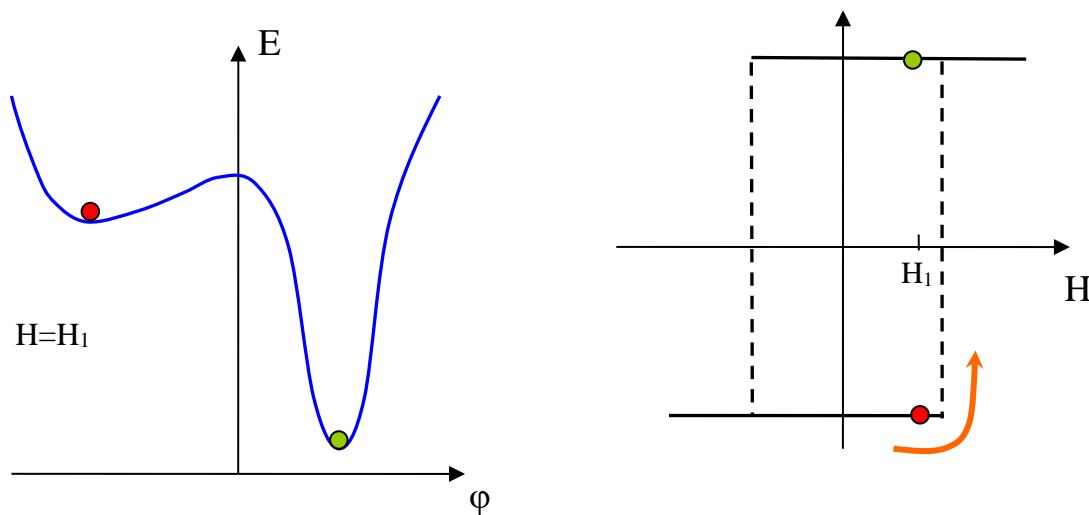
Thus we need to handle only 2 energy-term.

$$E = K \cdot V \cdot \sin^2 \varphi - mH \cos \varphi$$

The graph as a function of angle φ for different H (black line is for $H=0$):



One can see that the energy-minimum in the right side gradually becomes shallower for increasing field, but it still remains a local minimum. Then suddenly the system can come out of this potential-well and abruptly goes into the global energy-minimum.



This is a bistable system, appropriate for digital data storage. As we want to store more data, we need to decrease the size of the magnetic bits. However, not only the reading and writing process, but the preservation of data becomes problematic. This is because we are talking about **non-volatile** storage, which means that the data is required to be preserved when the storage device is not powered.

Superparamagnetism

<http://en.wikipedia.org/wiki/Superparamagnetism>

Superparamagnetism is a form of magnetism, which appears in small ferromagnetic or ferrimagnetic nanoparticles. In sufficiently small nanoparticles, magnetization can

randomly flip direction under the influence of temperature. In the absence of an external magnetic field, their magnetization appears to be in average zero: they are said to be in the superparamagnetic state. In this state, an external magnetic field is able to magnetize the nanoparticles, similarly to a paramagnet. However, their magnetic susceptibility is much larger than that of paramagnets.

Superparamagnetism occurs in nanoparticles which are single-domain, i.e. composed of a single magnetic domain. This is possible when their diameter is below 3–50 nm, depending on the materials. In this condition, it is considered that the magnetization of the nanoparticles is a single giant magnetic moment, sum of all the individual magnetic moments carried by the atoms of the nanoparticle. Those in the field of superparamagnetism call this “macro-spin approximation”.

Because of the nanoparticle’s magnetic anisotropy, the magnetic moment has usually only two stable orientations antiparallel to each other, separated by an energy barrier. If K is the nanoparticle’s magnetic anisotropy energy density and V its volume, then KV is the energy barrier. On the other hand, according to **equipartition theorem**, energy is shared equally among all of its various forms; for example,

the average energy per degree of freedom is $E = \frac{1}{2}kT$,

where k is the Boltzmann constant. However, as the magnetization of the particle is uniform (the individual moments always move coherently), the direction of the moment means only **ONE** degree of freedom, independently of the volume.

At finite temperature, there is a finite probability for the magnetization to flip and reverse the easy axis direction. The average time between two flips is called the Néel relaxation time

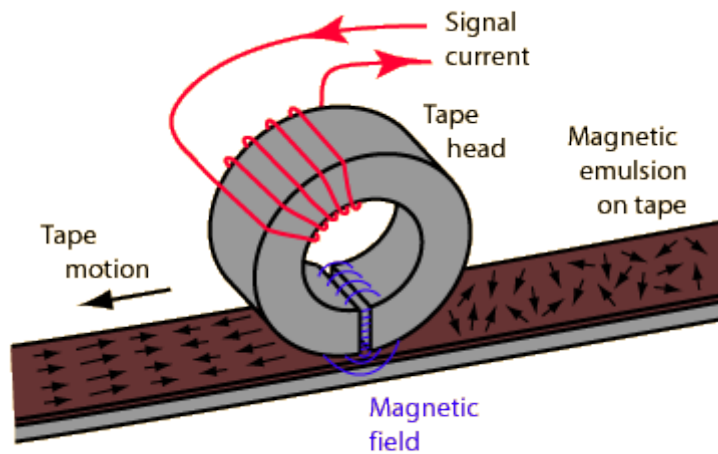
$$\tau \sim e^{\frac{KV}{kT}}$$

One have to note that superparamagnetism occurs below the Curie temperature of the material, thus the atomic moments locally ordered, but the aggregated moments of the particles have random orientation, therefore the average magnetization is zero.

Superparamagnetism sets a **limit on the storage density** of hard disk drives due to the minimum size of particles that can be used. This limit is known as the superparamagnetic limit.

Data storage in practice

Analog recording is based on the fact that remnant magnetization of a given material depends on the magnitude of the applied field. The magnetic material can be in the form of tape, with the tape in its blank form being initially demagnetized.



When recording, the tape runs at a constant speed and the electric current in the writing head produces magnetic field:

$$\oint \vec{H} d\vec{r} = I$$

The writing head magnetizes the tape with current proportional to the signal because of the torque exerted on the moments

$$\vec{\tau} = \vec{m} \times \vec{B}$$

or, equivalently, because of Zeeman energy.

$$E_p = -\vec{m} \cdot \vec{B}$$

A magnetization distribution is achieved along the magnetic tape. Finally, the distribution of the magnetization can be read out by electromagnetic induction,

$$\oint \vec{E} d\vec{r} = -\frac{d}{dt} \int \vec{B} d\vec{A}$$

reproducing the original signal.

These processes can be explained by classical physics (electromagnetism). However, the reason of the persistence of the memory is quantum-mechanical (exchange-interaction, anisotropy, etc.)

The magnetic tape is typically made by embedding magnetic particles in a plastic binder on polyester film tape. The commonly used magnetic particles are Iron oxide particles or Chromium oxide and metal particles with size of 0.5 micrometers. Analog recording was very popular in audio and video recording. In the last decades, however, tape recording has been gradually replaced by digital recording.

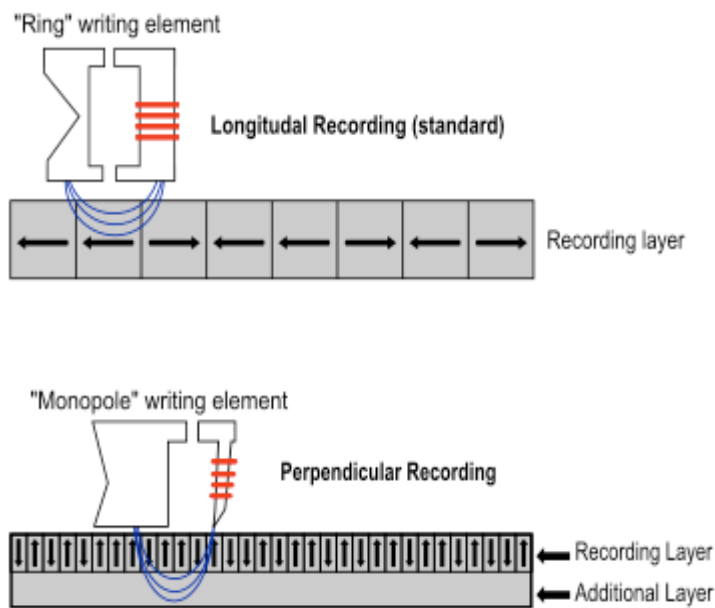
Digital recording:

Instead of creating a magnetization distribution in analog recording, digital recording only uses two stable, saturated magnetic states. Examples: floppy disks and hard disk drives (HDDs). Digital recording has also been carried out on tapes.

Recording media on HDDs use a stack of thin films to store information and a read/write head to read and write information to and from the media; various developments have been carried out in the area of used materials.

Older hard disk technology uses **longitudinal** recording. It has an estimated limit of 100 to 200 Gbit/in²

Current hard disk technology uses **perpendicular** recording, where the magnetization of the domains are perpendicular to the plane of the medium.



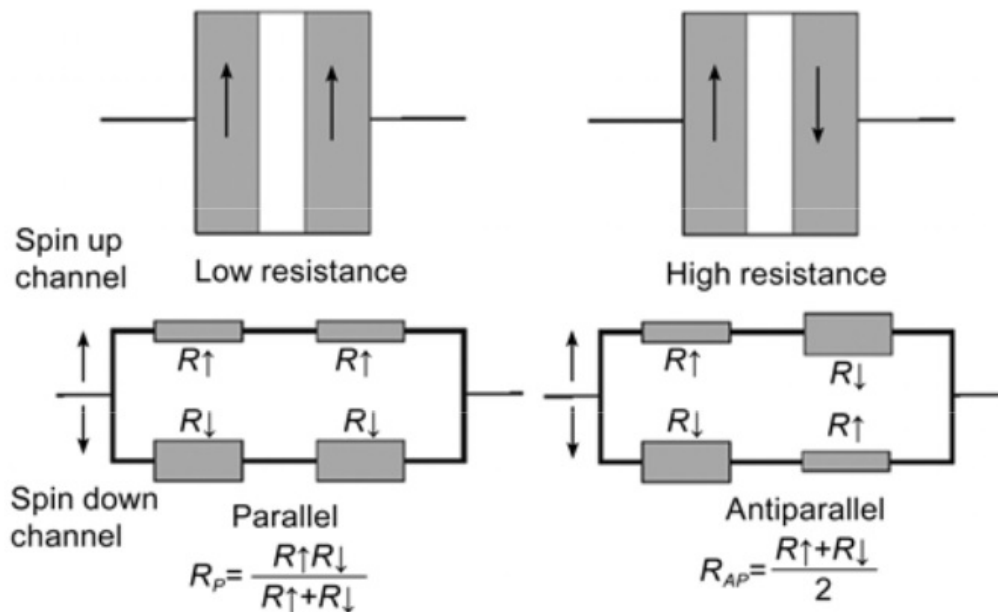
Example video:

<https://www.youtube.com/watch?v=wteUW2sL7bc>

GMR

Giant magnetoresistance (GMR) is a quantum mechanical magnetoresistance effect observed in thin-film structures composed of alternating ferromagnetic and non-magnetic conductive layers. The 2007 Nobel Prize in Physics was awarded to Albert Fert and Peter Grünberg for the discovery of GMR in 1988.

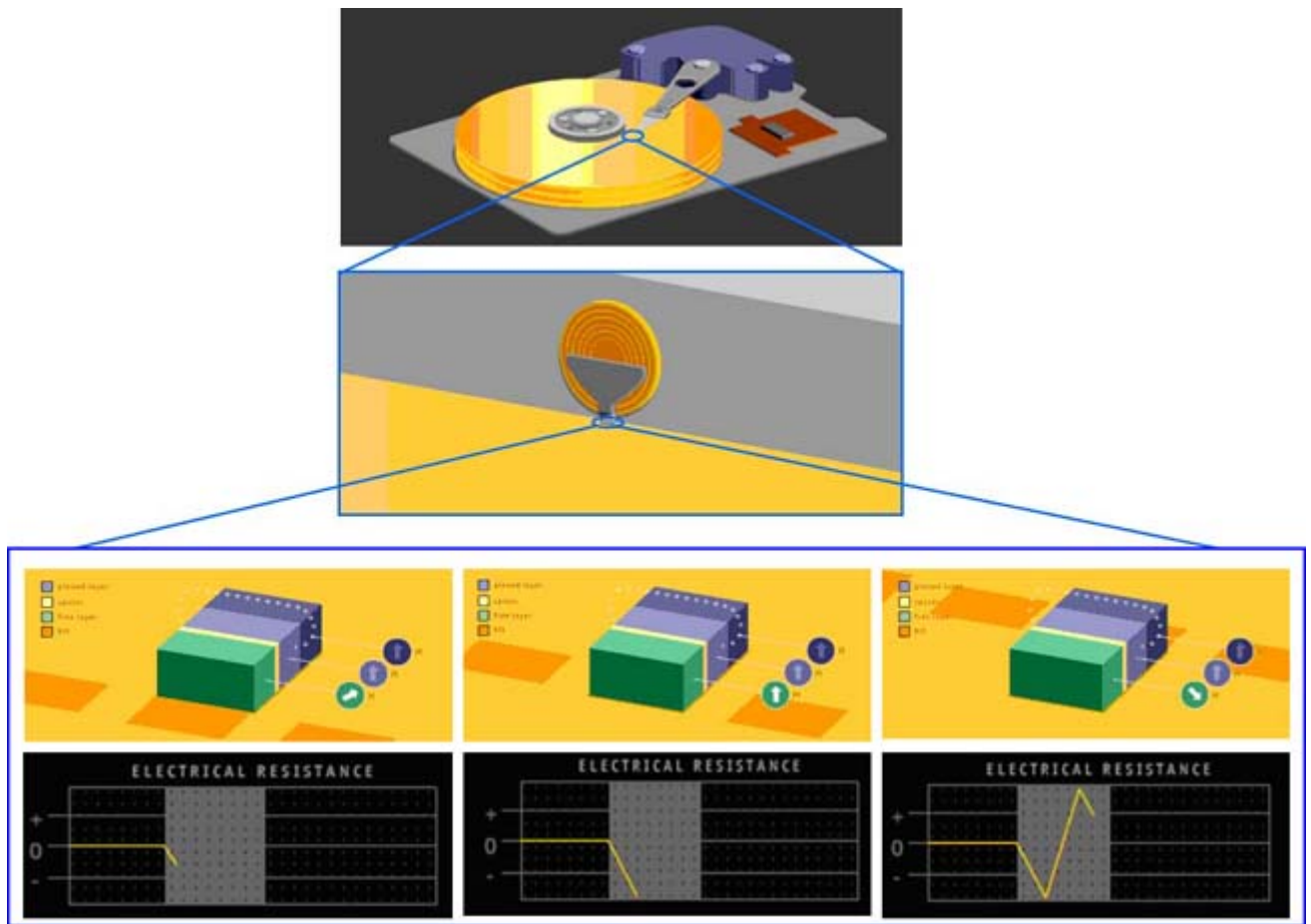
The effect is observed as a significant change in the electrical **resistance** depending on whether the magnetization of adjacent ferromagnetic layers are in a parallel or an antiparallel alignment. The overall resistance is relatively low for parallel alignment and relatively high for antiparallel alignment. The magnetization direction can be controlled, for example, by applying an external magnetic field. The effect is based on the dependence of electron scattering on the spin orientation.



The application of GMR are biosensors, microelectromechanical systems (MEMS) and other devices. However, the first and largest commercial success of giant magnetoresistance has been its use in the spin valve giant magnetoresistive sensor to read back information stored on hard disk drives.

The reading head is essentially a sandwich of two magnetic layers, a sensing (non-fixed) layer, a fixed (pinning) layer, and a non-magnetic layer between them. When the disk is rotating, the magnetic data creates changing magnetic field, which is an external magnetic field to the reading head, therefore deflects the magnetization in the sensing layer. When the field tends to align the magnetizations in the sensing and fixed layers, the electrical resistance of the sensor decreases, and vice versa. In order to let the sensing layer to freely change the orientation of its magnetization, one have to prevent strong RKKY interaction with the fixed layer.

Animation: <http://www.research.ibm.com/research/gmr.html>



In literature, the term giant magnetoresistance is sometimes confused with colossal magnetoresistance of ferromagnetic and antiferromagnetic semiconductors, which is not related to the multilayer structure and still not applied in data recording.

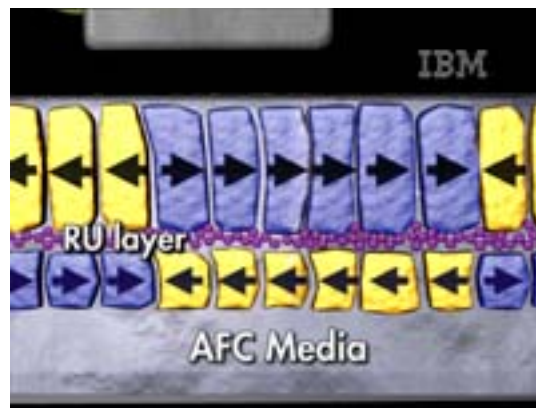
In this devices not only the charge, but the spin of the electrons plays an important role, therefore this research field is called spintronics instead of electronics.

AFC

Anti-Ferromagnetically Coupling, (2001)

This invention relates to very high areal recording density, anti-ferromagnetically coupled (“AFC”) magnetic recording media, such as hard disks, with enhanced RKKY-type coupling providing improved performance and data-density.

Conventional disk media stores data in only one magnetic layer, typically of a complex magnetic alloy (such as cobalt-platinum-chromium-boron, CoPtCrB). AFC media is a multi-layer structure in which two magnetic layers are separated by a three atom layer of ruthenium, a nonmagnetic metal. The precise thickness of the ruthenium causes the magnetization in each of the magnetic layers to be coupled in opposite or anti-parallel, directions by RKKY interaction which constitutes antiferromagnetic coupling.



Thus this method makes the potential well deeper in order to compensate for the smaller grain sizes. This is a solution to the problem of thermal instability (arising from the very small grain sizes) and the superparamagnetic effect. However, this approach is limited by the field provided by the writing head.

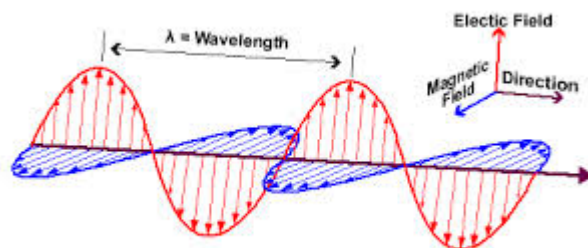
Magneto-optical recording

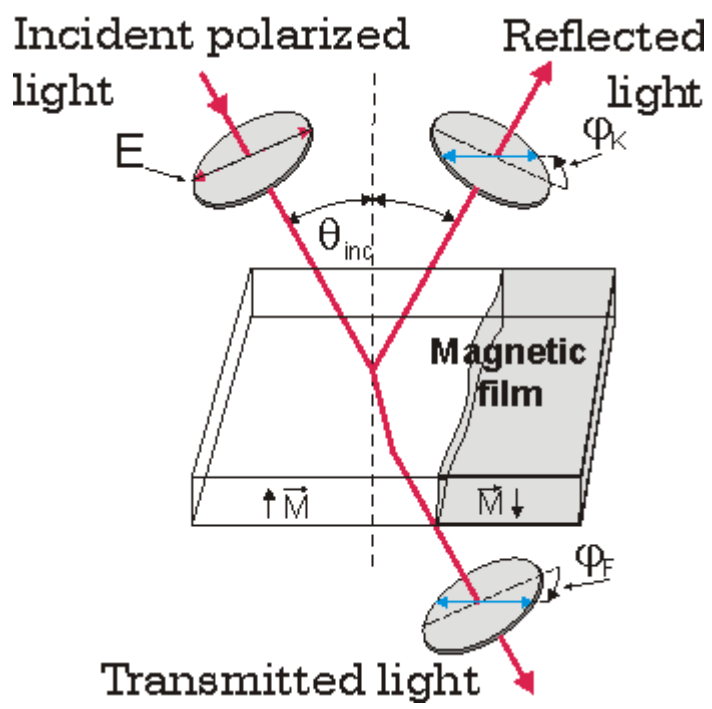
Magneto-optical recording writes/reads the data optically.

When **writing**, the magnetic medium is heated locally by a laser above its Curie temperature. It is heated only at the points where the state of the bit have to be changed. Then, a weak magnetic field is enough to switch the magnetization of the heated areas.

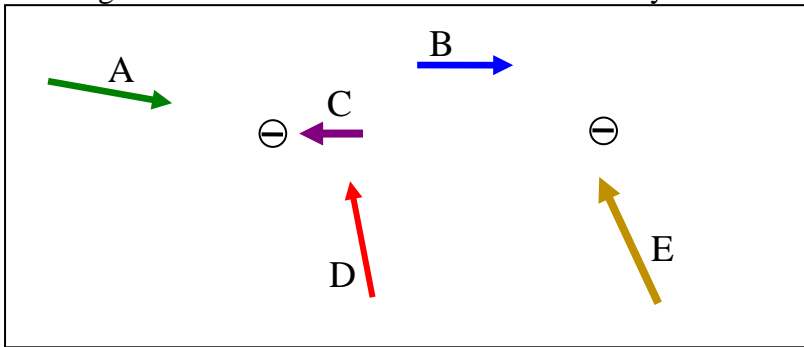
The **reading** process is based on magneto-optical **Kerr effect**. The laser is operated at a lower intensity (negligible heating), and emits polarized light. Kerr effect means that the interaction of polarized light with a magnetized material causes rotation of the plane of polarization. The angle of the rotation depends on the magnetic state of the material. The reflected light is analysed showing a noticeable difference between a 0 and 1.

The magnetic medium are typically amorphous R-FeCo thin film (R is a rare earth element). Magneto-optical recording is widespread only among the companies of the Far-East, e.g. in Japan (like Sony).





1. There are two identical negative point-charge on the plane. The arrows are supposed to sign the electric field. Which one is obviously incorrect?



2. Create true sentences by choosing on of the options in each brackets.

The $\left\{ \begin{array}{l} \text{north} \\ \text{south} \end{array} \right\}$ pole of a bar magnet $\left\{ \begin{array}{l} \text{attracts} \\ \text{repels} \end{array} \right\}$ a $\left\{ \begin{array}{l} \text{diamagnetic} \\ \text{paramagnetic} \end{array} \right\}$ material.

3. We have a bulk ferromagnetic material and $H=0$. We put the origin of our coordinate-system into the center of it. We observe that there is a 180° domain wall at the x-y plane. Above this plane, in the $z>0$ region, the magnetic moments point to the x direction, below the plane, in the $z<0$ region, they point to $-x$.

Now we start to increase H to the x direction. Which direction will the domain wall start to move?

A) x B) -x C) y D) -y E) z F) -z